LEARNING MATERIAL

STATE COUNCIL FOR TECHNICAL EDUCATION & VOCATIONAL TRAINING, ODISHA, BHUBANESWAR

THEORY OF MACHINE

(For Diploma and Polytechnic students)

4TH SEMESTER

CONTENTS

Chapter-1

Simple Mechanism

Syllabus:

1.1 Link, kinematic chain, mechanism, machine
1.2 Inversion, four bar link mechanism and its inversion
1.3 Lower pair and higher pair
1.4 Cam and followers

1.1 Understand the subject of theory of machine

Introduction :-

- A machine is a device by means of which available energy can be converted into derived form of useful work. It is the assembly of resistant bodies or links whose relative motions are successfully constrained so that available energy can be converted into useful work. Machines are used to transmit both motion and force

OR

A machine is device which receives energy and transforms it into some useful work.
Example: Lathe, Shaper, Scooter etc..

- Theory of machine is the branch of engineering which deals with the relative motion and force between various machine elements.

Resistance body:

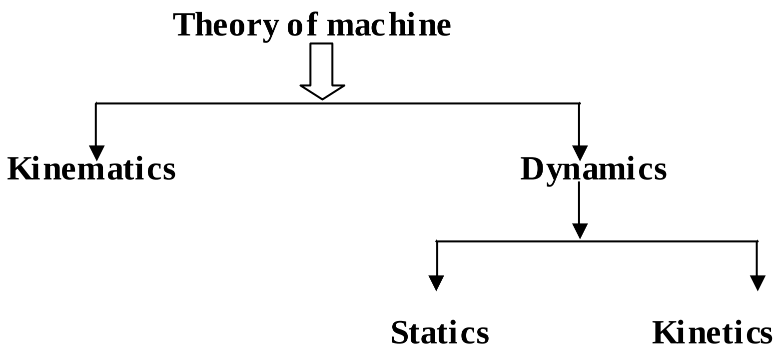
- A body is said to be resistant body if it is capable of transmitting the required force with negligible deformation. These bodies are the parts of the machines which are used for transmitting motion and forces.
- A link need not necessary be a rigid body, but it must be a resistant body.

  Example: Springs, belts and oil used in hydraulic press are not rigid links but they are resistant bodies.
- Structure is an assemblage of a number of resistant bodies having no relative motion between them. Structures are meant for taking loads

Difference between machine and structure:

Machine	Structure
• There is relative motion between its members. • It converts available energy into useful work. • Members are meant to transmit motion and forces. • Example : car, lathe etc..	• There is no relative motion between them. • It does not convert the available energy into useful work. • Members are meant to take up loads only. • Example : bridge, frames, buildings etc..

Classification of theory of machine:



Kinematic:

Kinematic of machines is that branch of theory of machines which deals with the study of relative motion of parts of which the machines are constituted, neglecting consideration of forces producing it.

Dynamics:

Dynamics of machine is that branch of theory of machines which deals with study of forces acting on different parts of the machine.

Statics:

Statics deals with the study of forces acting on the various parts of machines (assumed to be without mass), when they are at rest.

Kinetics:

Kinetics deals with the study of forces which are produced due to inertia of moving parts of machines.

OR

Kinetics deals with the study of inertia forces which are produced by the combined effect of the mass and the motions of the various parts.

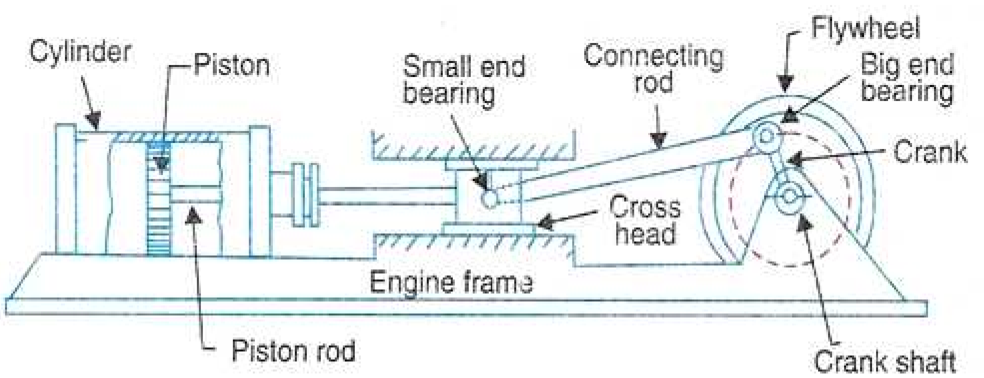
Kinematics:

Kinematics of motion i.e. the relative motion of bodies without consideration of the forces causing the motion.

Plane motion

Rectilinear/translatory (Straight line) | **curvilinear motion (Curved path)**

Kinematic link:



Reciprocating Steam Engine:

Each part of a machine, which moves relative to ssome other part, is known as link or element.

Links in reciprocating engine

Link-1 : piston, piston rod and cross head

Link-2 : crank, crankshaft and flywheel

Link-3 : cylinder, engine frame and main bearings

Link should have two characters.

1. It should have relative motion.
2. It should be a resistant body.

Types of links :

1. **Rigid link :**

   A rigid link is on which does not undergo any deformation while transmitting motion.
   Example : Deformation of a connecting rod, crank etc.. is not appreciable.

2. **Flexible link :**

   A flexible link in one which is partly deformed in a manner not to affect the transmission of motion.
   Example : belts, ropes, chains and wires

3. **Fluid link :**

   A fluid link is one which is formed by having a fluid in a reciprocal and the motion is transmitted through the fluid by pressure or compression only as in the case of hydraulic presses, jacks and brakes.

Kinetic pair:

The two links or elements of a machine, when in contact with each other are said to form a pair.

When two elements or links are connected in such a way that their relative motion is constrained they form a kinematic pair and the process of connecting them is called pairing.

Three types of constrained motions :

1. **Completely constrained motion**
2. **Incompletely constrained motion**
3. **Successfully constrained motion**

1. **Completely constrained motion :**

   When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.
   Example : The piston and cylinder(in a steam engine) from a pair and the motion of the piston is limited to a definite direction.

2. **Incomplete constrained motion :**

   when the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained. The change in the direction of impressed force may after the direction of relative motion between the pair.

Example : circular shaft in a circular hole may rotate or slide.

3. **<u>Successfully constrained motion :</u>**

When the motion between the elements, forming a pair is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion.

Example : shaft in a foot step bearing

The shaft may rotate in the bearing or it may move upwards. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is successfully constrained.

Kinematic pair classification:

1. According to the type of relative motion between the elements.
2. According to the type of contact between the elements.
3. According to the type of closure.

Kinematic pairs

Relative motion between the element	Contact between the element	Types of closure
(i) sliding pair	(i) lower pair	(i) self closed pair
(ii) turning pair	(ii) higher pair	(ii) force-closed pair
(iii) rolling pair		
(iv) screw pair		
(v) spherical pair		

<u>Lower pair :</u>

When the two elements of a pair have a surface contact while relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair.

Example : sliding pair, turning pair, piston cross head, tailstock on the leather bed, shaft revolting in a bearing, steering gear, universal joint.

The relative motion is purely sliding or turning.

<u>Higher pair:</u>

When the two elements of a pair have a line or point contact while in motion, the pair so formed is known as a higher pair. The relative motion being the combination of sliding and turning, thus it is complex.

Example : belt, rope and chain drives, gears, cam and follower, ball and roller bearing.

<u>Kinematics chain:</u>

When kinematic pair are so coupled that the last link is joined to the first link to transmit a definite completely unsuccessfully constrained motion- it forms a kinematic chain.

If each link is assumed to form two parts, with two adjacent links, then the rotation.

$$L = 2p - 4 \quad \text{-------(i)}$$

J = [illegible] – 2 -------(ii)

L → no. of links

p → no. of pairs

J → no. of joints

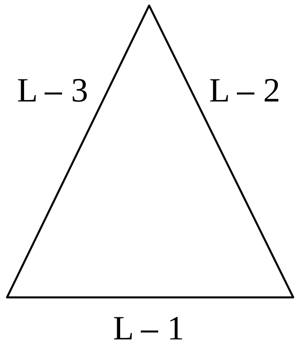
Both equation (i) & (ii) are applicable to kinematic chain with lower pairs. Also applicable to kinematic chain with higher pairs. Higher pair is equivalent to two lower pairs with an additional element or link.

<u>Problem-1</u>

L = 3

p = 3

j = 3



L = 2p – 4

⇨ 3 = 2* 3 – 4

⇨ 3 > 2

⇨ LHS > RHS

J = $\frac{3}{2}L$ – 2

⇨ 3 = $\frac{3}{2}$* 3 – 2

⇨ 3 > 2.5

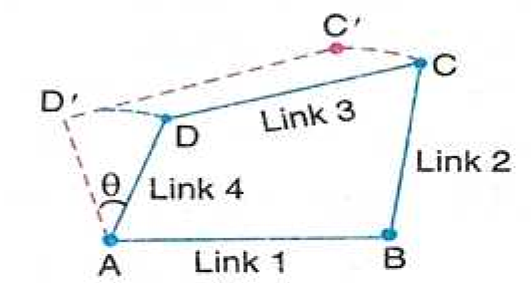
As LHS > RHS, it is not a kinematic chain and hence number of relative motion is possible. Such type of chain is called locked chain & forms a liquid frame or structure.

<u>Problem-2</u>

l = 4

p = 4

j = 4



l = 2p – 4

⇨ 4 = 2* 4 – 4

⇨ 4 = 4

⇨ LHS = RHS

J = $\frac{3}{2}L$ – 2

⇨ 4 = $\frac{3}{2}$* 4 – 2

⇨ 4 = 4

⇨ LHS = RHS

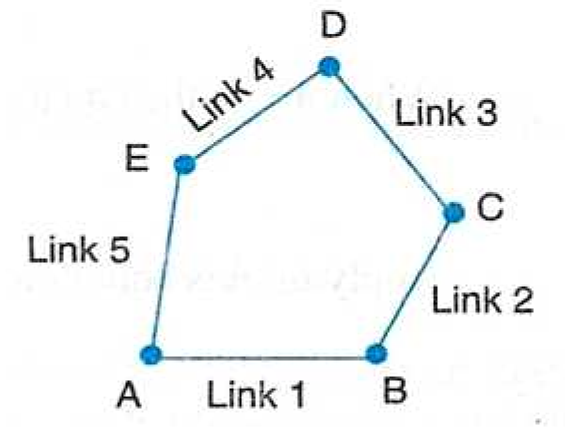
It is a kinematic chain as LHS = RHS also called as constrained kinematic chain. Since link CD can move in a definite direction. It is constrained.

Problem-3

L = 5

P = 5

J = 5



L = 2p – 4

⇨ 5 = 2* 5 – 4

3

⇨ 5< 6

⇨ LHS < RHS

J = [illegible]L – 2

⇨ 5 = [illegible]* 5 – 2

⇨ 5 < 5.5

⇨ LHS < RHS

So it is not a kinetic chain. This chain is called unconstrained chain. The relative motion is not completely constrained. This type of chain is used rarely.

<u>Mechanism:</u>

When one element or link of a kinematic chain is fixed, the arrangement may be used for transmitting or transforming motion. It is termed as mechanism.

Types of mechanism

Simple mechanism | Compound mechanism

<u>Simple mechanism:</u>

Mechanism having four links.

Example: Coupling rod mechanism of a locomotive beam engine etc.

<u>Compound mechanism:</u>

Mechanism having more than four lines. It may be formed combining two or more simple mechanism.

<u>Inversion :</u>

Different mechanisms can be obtained by fixing different links in a kinematic chain. It is known as inversion of the original kinematic chain.

Suppose number of links of a kinematic chain = L

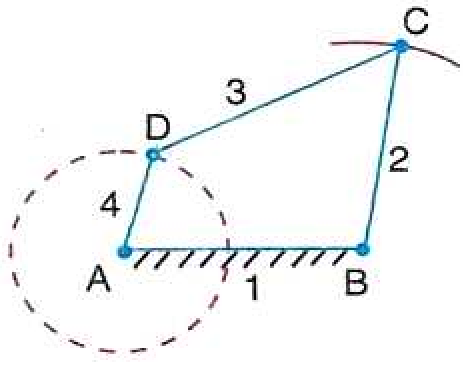
So L different mechanisms may be obtained by fixing each of the lines in turn. Each mechanism is termed as inversion.

<u>Types of kinematic chains(most important kinematic chain)</u>

1. **Four bar chain or quadric cycle chain**
2. **Single slider crank chain**
3. **Double slider crank chain**

<u>Four bar chain or Quadric cycle chain:</u>

- The kinematic chain is a combination of four limks or four kinematic pairs, such that the relative motion between the links or elements is completely constrained.
- Four links A, B, C & D each of them form a turning pair.



Grashof's law:

According to Grashof's law for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

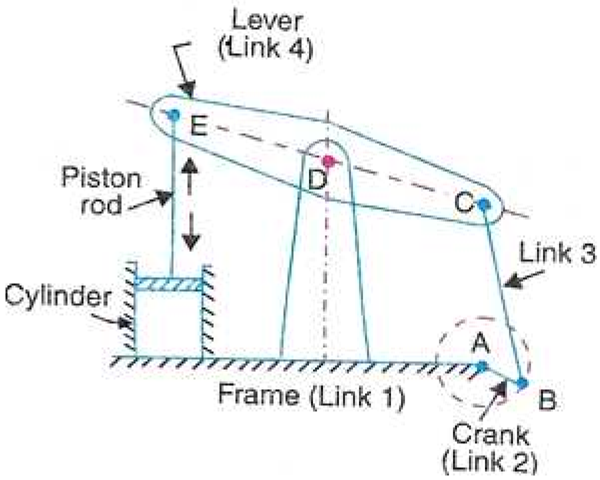
Inversions of four bar mechanism:

1. **Beam engine**
2. **Coupling rod of a locomotives (Double crank mechanism)**
3. **Watt's indicator mechanism (Double lever mechanism)**

Link description:

- In a four bar chain the mechanism in which if no link makes a complete revolution will not be useful.
- In four bar chain, one of the links in particular the shortest link will make a complete revolution relative to the other three links, if it satisfies the Grashof's law. Such a link is known as crank or driver (i.e. AB)
- Link BC which makes a partial rotation or oscillation is known as lever or rocker or follower.
- The link CD which connects the crank and lever is called connecting rod or coupler.
- The fixed link AB is known a frame of the mechanism.

Beam engine (Type of steam engine) :



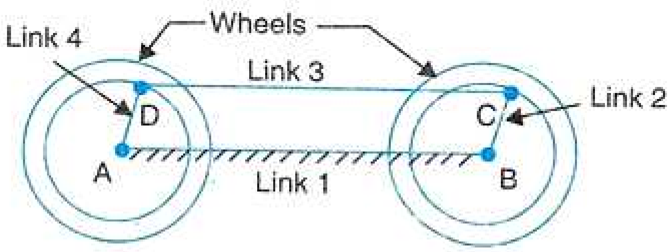
Beam Engine

- Here, when the crank rotates about the fixed center A, the lever oscillates about the fixed center D.
- The end E of CDE is connected to the piston which reciprocates as a result.

Rotary motion ------> Reciprocating motion --------> To power steam ship

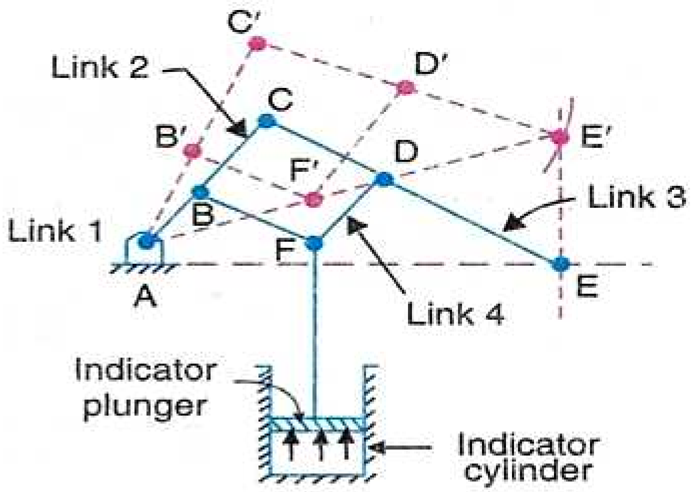
Coupling rod of a locomotive (Double crank mechanism):

- There AD = BC (Both crank) connected to the respective wheels.
- CD is called connecting rod.
- AB is fixed to maintain constant centre to centre distances between the wheels.
- This mechanism is meant for transmitting rotary motion from one wheel to the other.



Coupling of a locomotive

Watt's Indicator Mechanism:



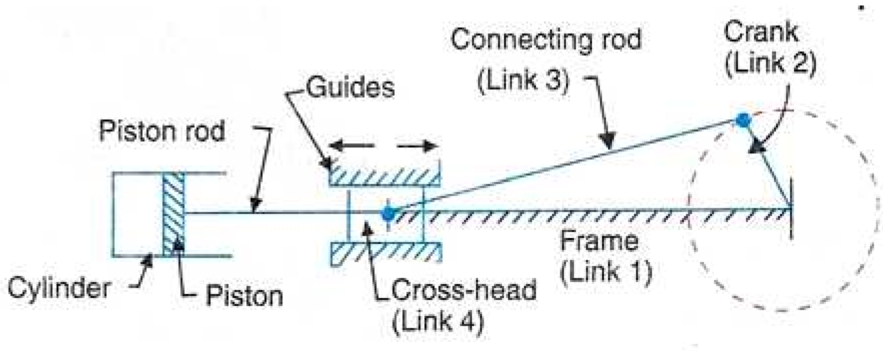
Watt's Indicator Mechanism

- Four links :
  Link-1 : fixed, link-2 - (AC), link-3 - (CE), and link – 4 (BDF)
- The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator ________.
- For small displacement of the mechanism, the tracing point E traces out approximately a straight line.

Single Slider Crank Chain:

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is, usually, found in reciprocating motion and vice versa.

In a single slider crank chain, as shown in figure the links 1 and 2, 2 and 3, and links 3 and 4 from three turning pairs while the links 4 and 1 from a sliding pair.



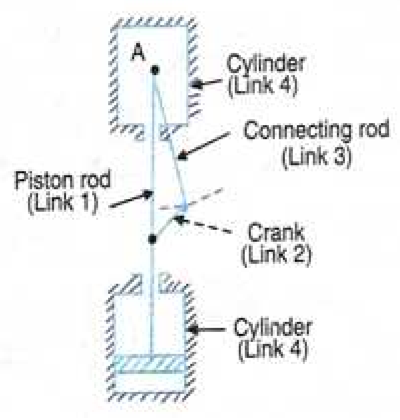
The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank ; links 3 corresponds to the connecting rod and links 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

Inversions of single slider crank chain

We have seen in the previous article that a single slider crank chain is a four-link mechanism. We know that by fixing, in turn, different links in a kinematic chain, an inversion is obtained and we can obtain as many mechanisms as the links in a kinetic chain. It is thus obvious, that four inversions of a single slider crank chain are possible. These inversions are found in the following mechanisms.

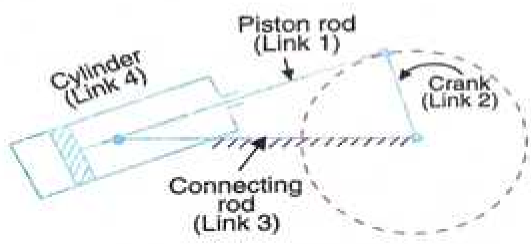
1. pendulum pump or bull engine:

In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair), as shown in figure. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in figure.



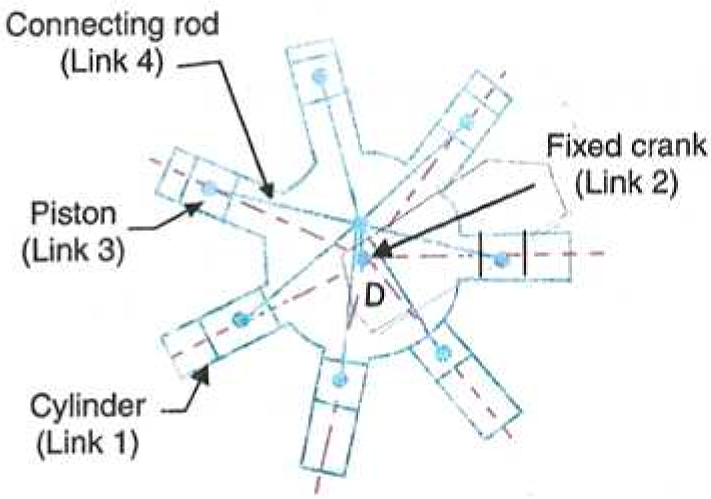
2. Oscillating cylinder engine:

The arrangement of oscillating cylinder engine mechanism, as shown in figure, is used to convert reciprocating motion into rotary motion. In this mechanism, thye link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotats, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

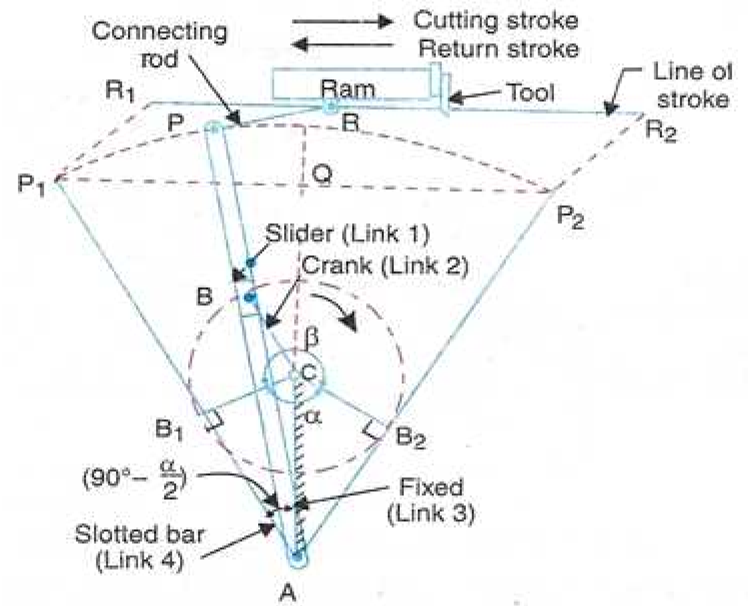


3. rotary internal combustion engine or gnome engine:

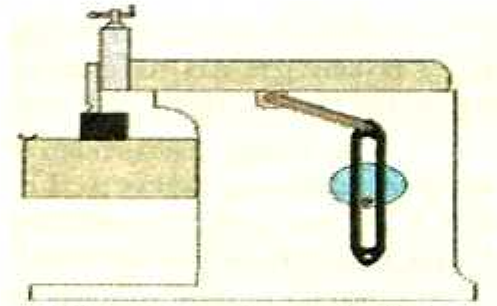
Some time back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre D,as shown in figure, while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates the piston (link 3) reciprocates inside the cylinders forming link 1.



In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in figure. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R_1R_2. The line of stroke of the ram (i.e. R_1R_2) is perpendicular to AC produced.



In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_2 to CB_1 (or through an angle β) in the clockwise direction.the return stroke occurs when the crank rotates from the position CB_2 to CB_1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed,therefore,



The Shaping Machine

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \quad \text{or} \quad \frac{360° - \alpha}{\alpha}$$

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1 AQ$$

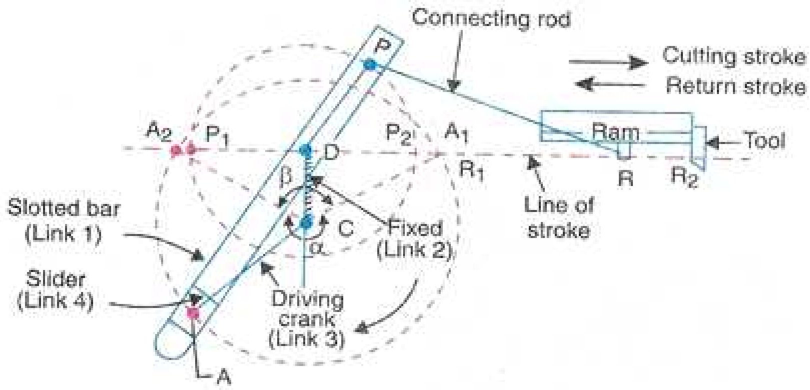
$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos \frac{\alpha}{2} \qquad \text{.....} (\because AP_1 = AP)$$

$$= 2AP \times \frac{CB_1}{AC} \qquad \cdots \left(\because \cos \frac{\alpha}{2} = \frac{CB_1}{AC}\right)$$

$$= 2AP \times \frac{CB}{AC} \qquad \ldots \ (\because CB_1 = CB)$$

5. Whitworth quick return motion mechanism:

This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD(link 2) forming the turning pair is fixed, as shown in figure. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. the connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, i.e. along a line passing through D and perpendicular to CD.



When the driving crank CA moves from the position CA_1 to CA_2 (or the link DP from the position DP_1 to DP_2) through an angle β in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2 PD.

Now when the driving crank moves from the position CA_2 to CA_1 (or the link DP from DP_2 to DP_1) through an angle α in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram(i.e. during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Similarly, the time taken during the right to left movement of the ram(or during the idle or return stroke) will be equal to the time taken by the driving crank to move from CA_2 to CA_1

Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke(or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

[illegible]

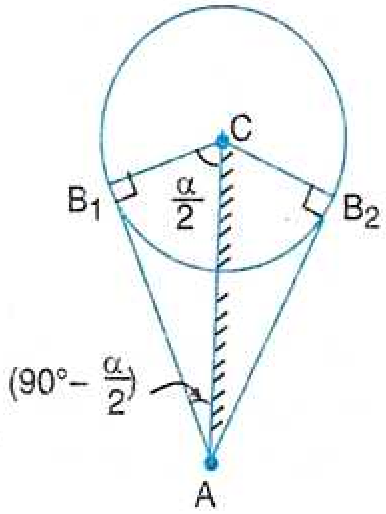
Example:

A crank and slotted lever mechanism used in a shaper has a centre distance of 300 mm between the centre of oscillation of the slotted lever and the centre of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke.

Solution:

Given: AC = 300 mm ; CB_1 = 120 mm

The extreme positions of the crank are shown in figure. We know that



$$\sin \angle CAB_1 = \sin(90° - \alpha/2)$$

$$= \frac{CB_1}{AC} = \frac{120}{300} = 0.4$$

$$\therefore \quad \angle CAB_1 = 90° - \alpha/2$$

$$= \sin^{-1} 0.4 = 23.6°$$

or $$\alpha/2 = 90° - 23.6° = 66.4°$$

and $$\alpha = 2 \times 66.4 = 132.8°$$

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha} = \frac{360° - 132.8°}{132.8°} = 1.72 \textbf{ Ans.}$$

Example:

In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and time ratio of cutting stroke to the return stroke.

If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

Solution:

Given: AC = 240 mm ;

CB_1 = 120 mm ;

AP_1 = 450 mm

Inclination of the slotted bar with the vertical

Let [illegible] inclination of the slotted bar with the vertical.

The extreme position of the crank are shown in figure. We know that

$$\sin \angle CAB_1 = \sin\left(90° - \frac{\alpha}{2}\right)$$

$$= \frac{B_1C}{AC} = \frac{120}{240} = 0.5$$

$$\therefore \angle CAB_1 = 90° - \frac{\alpha}{2}$$

$$= \sin^{-1} 0.5 = 30° \text{ Ans.}$$

Time ratio of cutting stroke to the return stroke

We know that

$$90° - \alpha / 2 = 30°$$

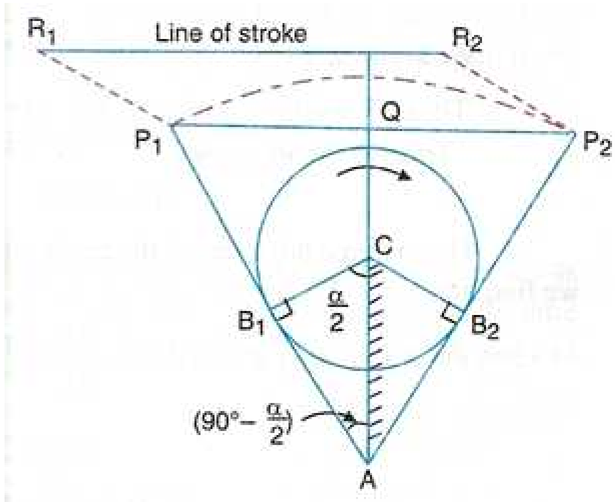
$$\therefore \quad \alpha / 2 = 90° - 30° = 60°$$

or $$\alpha = 2 \times 60° = 120°$$

$$\therefore \quad \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha}$$

= [illegible]

= 2



Length of the stroke:

We know that length of the stroke,

$$R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin(90^0 - \text{[illegible]}$$

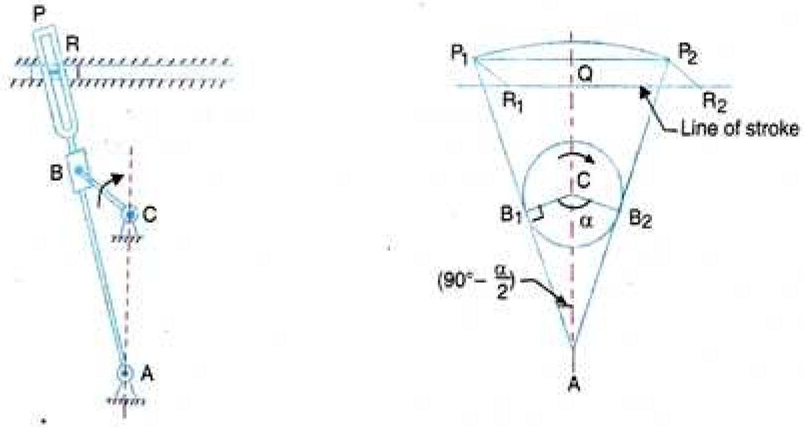
$$= 2 \text{ [illegible]}$$

[illegible]

Example:

Figure shows the layout of a quick return mechanism of the oscillating link type, for a special purpose machine. The driving crank BC is 30 mm long and time ratio of the working stroke to the return stroke is to be if the length of the following stroke of R is 120 mm, determine the dimensions of AC and AP.

Solution:



Given: BC = 30 mm ;
R_1R_2 = 120 mm ;

We know that

$$\frac{\text{Time of working stroke}}{\text{Time of return stroke}} = \frac{360 - \alpha}{\alpha} \quad \text{or} \quad 1.7 = \frac{360 - \alpha}{\alpha}$$

$\therefore \quad \alpha = 133.3° \quad \text{or} \quad \alpha/2 = 66.65°$

The extreme positions of the crank are shown in Fig. 5.31. From right angled triangle AB_1C, we find that

$$\sin(90° - \alpha/2) = \frac{B_1C}{AC} \quad \text{or} \quad AC = \frac{B_1C}{\sin(90° - \alpha/2)} = \frac{BC}{\cos \alpha/2}$$

$$\ldots (\because B_1C = BC)$$

$$\therefore \quad AC = \frac{30}{\cos 66.65°} = \frac{30}{0.3963} = 75.7 \text{ mm } \textbf{Ans.}$$

We know that length of stroke,

$$R_1R_2 = P_1P_2 = 2P_1Q = 2\,AP_1 \sin(90° - \alpha/2) = 2\,AP_1 \cos \alpha/2$$

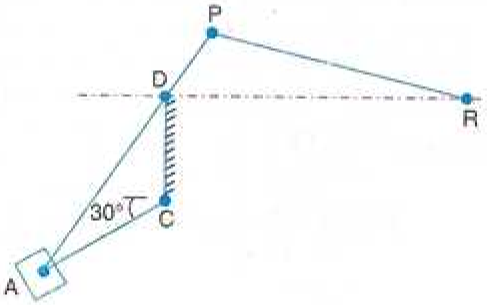
$$120 = 2\,AP \cos 66.65° = 0.7926\,AP \quad \ldots (\because AP_1 = AP)$$

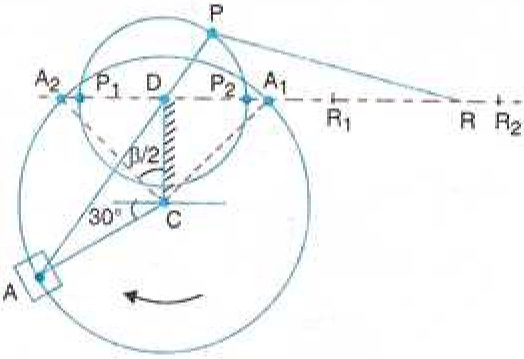
$$\therefore \quad AP = 120/0.7926 = 151.4 \text{ mm } \textbf{Ans.}$$

Example:

In a whitworth quick return motion mechanism, as shown in figure, the distance between the fixed centres is 50 mm and the length of the driving crank is 75 mm. The length of the slotted lever is 150 mm and the length of the connecting rod is 135 mm. Find the ratio of the time of cutting stroke to the time of return stroke and also the effective stroke.

Solution. Given : CD = 50 mm ; CA = 75 mm ; PA = 150 mm ; PR = 135 mm





The extreme positions of the driving crank are shown in Fig. From the geometry of the figure,

$$\cos\beta/2 = \frac{CD}{CA_2} = \frac{50}{75} = 0.667 \qquad \ldots (\because CA_2 = CA)$$

$$\therefore \qquad \beta/2 = 48.2° \quad \text{or} \quad \beta = 96.4°$$

Ratio of the time of cutting stroke to the time of return stroke

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360-\beta}{\beta} = \frac{360-96.4}{96.4} = 2.735 \text{ Ans.}$$

Length of effective stroke

In order to find the length of effective stroke (*i.e.* R_1R_2), draw the space diagram of the mechanism to some suitable scale, as shown in Fig. 5.33. Mark $P_1R_1 = P_2R_2 = PR$. Therefore by measurement we find that,

Length of effective stroke = R_1R_2 = 87.5 mm **Ans.**

Double slider crank chain:

A kinematic chain which consists of the turning pair and two sliding pair is known as **double slider crank chain,** as shown in figure.we see that the link 2 and link 1 form one turning pair and link 2 and link 3 from the second turning pair. The link 3 and link 4 from one sliding pair and link 1 and link 4 from the second sliding pair.

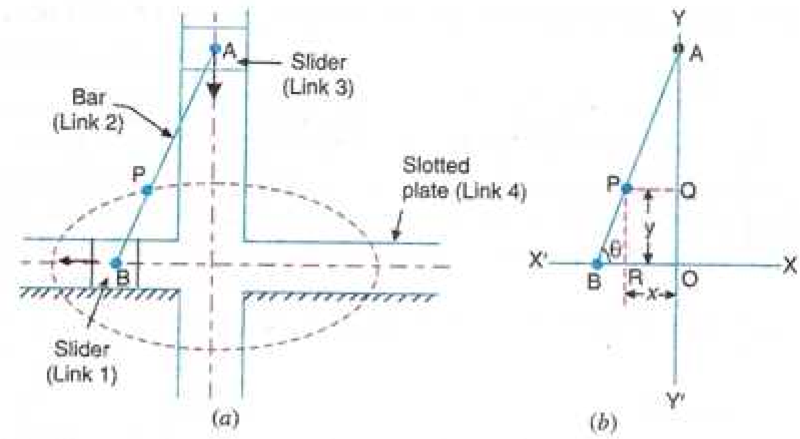
Inversions of double slider crank chain:

The following three inversions of a double slider crank chain are important from the subject point of view:

1. Elliptical trammels:

It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shows in figure. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and from sliding pairs with link 4. The link AB (link 2) is a bar which forms turning pairs with links 1 and 3.

When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as P traces out an ellipse on the surface of link 4, as shown in figure (a). A little consideration will show that AP and BP are semi-major axis and semi-minor axis of the ellipse respectively. This can be proved as follows:



Let us take OX and OY as horizontal and let the link BA is inclined at an angle θ with the horizontal, as shown in figure (b). Now the co-ordinates of the point P on the link BA will be

$$x = PQ = AP\cos\theta; \qquad \text{and } y = PR = BP\sin\theta$$

or $$\frac{x}{AP} = \cos\theta; \qquad \frac{y}{BP} = \sin\theta$$

squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

This is the equation of an ellipse. Hence the path traced by point P is an ellipse whose semi-major axis is AP and semi-minor axis is BP.

If P is the mid- point of link BA, then AP = BP. The above equation can be written as

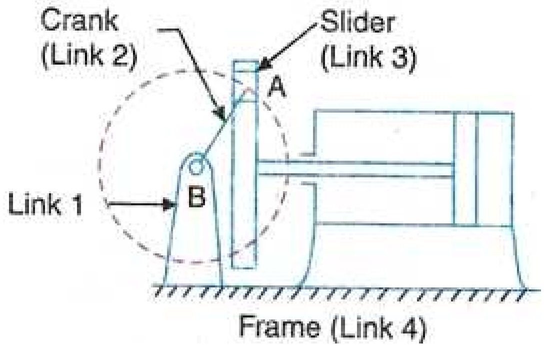
$$\frac{x^2}{(AP)^2} + \frac{y^2}{(AP)^2} = 1$$

$$\text{or } x^2 + y^2 = (AP)^2$$

This is the equation of circle whose radious is AP. Hence if P is the mid-point of link BA, it will trace a circle.

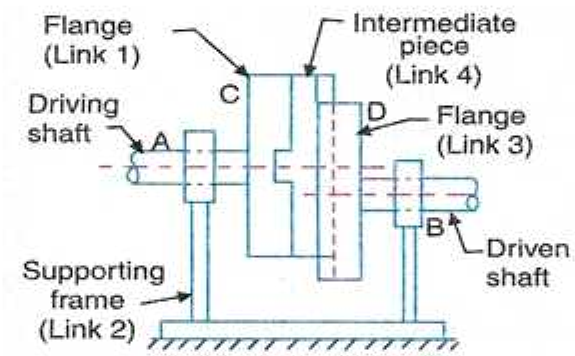
2. Scotch yoke mechanism:

This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In figure, link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about B as centre, the link 4(which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.



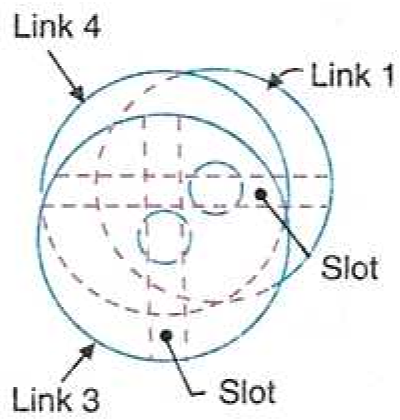
3. Oldham's Coupling:

An Oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shows in figure (a). The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.

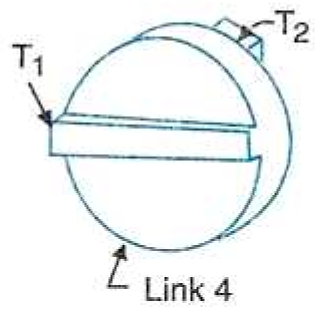


(a)

The link 1 and link 3 from turning pairs with link 2. These flanges have diametrical slots cut in their inner faces, as shown in figure. The intermediate piece (link 4) which is a circular disc, have two tongues(i.e. diametrical projections) T_1 and T_2 on each face at right angles to each other, as shown in figure (c). the tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.



(b)



(c)

When the driving shaft A is rotated, the flange C (link 1) causes the intermediate piece (link 4) to rotate at the same angle through which the flange has rotated, and it further rotates the flange D (link 3) at the same angle and thus the shaft B rotates. Hence link 1,3 and 4 have the same angular velocity at instant. A little consideration will show, that there is a sliding motion between the link 4 and each of the other link 1 and 3.

If the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of diameter equal to the distance between the axes of the two shafts. Therefore, the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

Let: = angular velocity of each shaft in rad/s, and

d = distance between the axes of the shafts in metres.

 maximum sliding speed of each tongue (in m/s),

v = G

CAM AND FOLLOWERS:

Definition:

A cam may be defined as a rotating, oscillating or reciprocating body which imparts a reciprocating or oscillating motion to another body, called a follower.

The cam and the follower constitute a higher pair.

Application:

Used in clocks, printing machines, internal combustion engines for operating valves, paper cutting machines, automatic screw cutting machines, shoe making machinery, feed mechanism of automatic lathe.

Classification of follower:

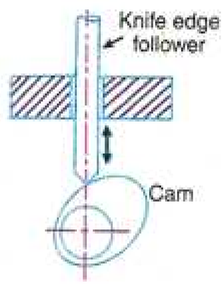
Follower

Surface in contact	Motion of the follower	Path of motion of the follower
(i) Knife edge follower	(i) Reciprocating or translating follower	(i) Radial follower
(ii) Roller follower	(ii) Oscillating or rotating follower	(ii) Offset follower
(iii) Flat faced or		
(iv) Spherical faced follower		

Manufacturing of cam :

A cam is difficult to manufacture especially when it is to be produced in small quantities. When produced on a mass scale, it is manufactured by a punch press, by die casting or by milling from the master cam.

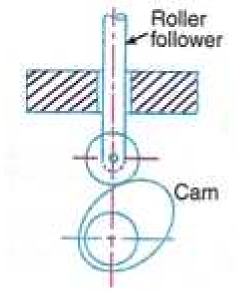
Knife edge follower :



- When the connecting end of the follower has a sharp knife edge, it is called a knife edge follower.
- Seldom used.
- Small area of contacting results in excessive wear.

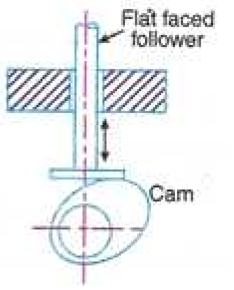
- A considerable side thrust exists between the follower and the guide.

Roller follower :

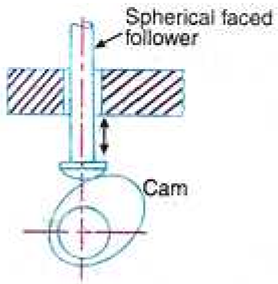


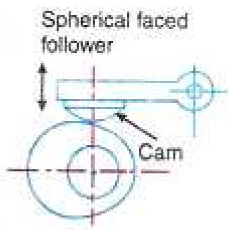
- When the connecting end of the follower is a roller, it is called a roller follower.
- Since rolling motion between the contact surfaces, the rate of wear is greatly reduced.
- Also side thrust exist between the follower and the guide.
- Extensively used (where more space is available)
- Application – Stationary gas and oil engines, aircraft engines

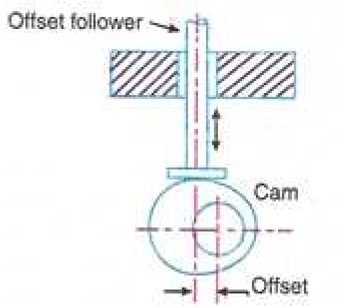
Flat faced or mushroom follower :



Spherical faced follower :







Note:

When a flat faced follower is used in automobile engine, high surface stresses are produced as to minimize these stresses, the flat end is machined to a spherical shape.

Reciprocating or translating follower:

When the follower reciprocates in guides as the cam rotates uniformly, it is known reciprocating follower.

Oscillating or rotating follower :

When the uniform rotary motion of the cam is connected into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower.

Radial follower:

When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower.

Offset follower:

- When the motion of the follower is along an axis away from the axis. If cam centre, it is called off-set follower.
- Off-set can be done by springs, gravity or hydraulic means.

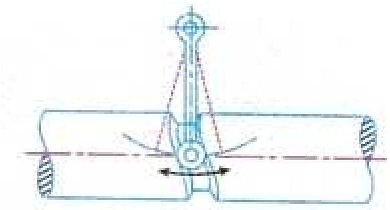
Classification of cam :

Cams

Radial or Disc cam **Cylindrical cam**

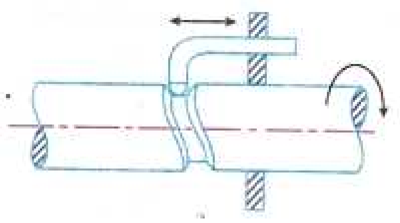
Reciprocating cam **Tangent cam** **Circular cam**

Radial or Disc cam :



In radial or disc cam, the follower reciprocates or oscillates in a direction perpendicular to the axis of cam rotation or perpendicular to the cam shaft.

Cylindrical cam :



- In cylinder cam, the follower reciprocates or oscillates in a direction perpendicular to the cam shaft.
- The follower rides in a groove at its cylindrical surfaces.

Chapter-2

Friction

Syllabus:

2.1 Revision of topic previously taught
2.2 Friction between nut and screw for square thread, screw jack
2.3 Bearing and its classification, Description of roller, needle roller & ball bearings.
2.4 Torque transmission in flat pivot & conical pivot bearings.
2.5 Flat collar bearing of single and multiple types.
2.6 Torque transmission for single and multiple clutches
2.7 Working of simple frictional brakes.
2.8 Working of Absorption type of dynamometer

DEFINITION:

When a body moves or tends to move on another body (surface/block), there exists some resistance or opposing force in a direction opposite to the direction of the movement of the body. This opposing force is called force of friction or friction.

TYPES OF FRICTION:

i. **Dry friction**
ii. **Rolling friction**
iii. **Skin friction**
iv. **Film friction**

(i) Dry friction:

This type of friction takes place between two bodies having relative sliding motion, when there is no intermediate fluid between their surfaces.

(ii) Rolling friction :

This type of friction occurs between two bodies directly in contact when the relative motion between them is purely rolling as in case of roller & ball bearing.

(iii) Skin friction :

When the two bodies between which the relative motion is separated by a film or lubricant of infinitesimally small thickness then the friction is called skin friction.

(iv) Film friction :

When the surfaces of two bodies are completely separated by a film of lubricant then that friction is called film friction.

Laws of friction:

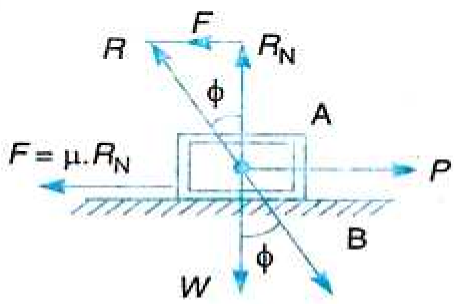
- When two bodies are in contact, the force of friction always acts in a direction opposite to that in which the body moves or tends to move.
- The force of friction is dependent upon the type of material of the bodies whose surfaces are in contact.
- The force of friction is independent of the area of contact between the two surfaces.
- The force of friction is independent of the relative velocity of sliding between the two surfaces.
- The force of frictions directly proportional to the normal reaction between the surfaces in contact for a particular material in contact for a particular material of which the bodies are made up.

$$F \propto N$$

$$\Rightarrow F = \mu N$$

Where μ = proportionality constant. i.e. also called as co-efficient of friction.

Limiting Angle of Friction:



$F = \mu N$

⇨ $\mu =$

$\tan \varphi =$

⇨ $\tan \varphi = \mu$

Liming angle of friction

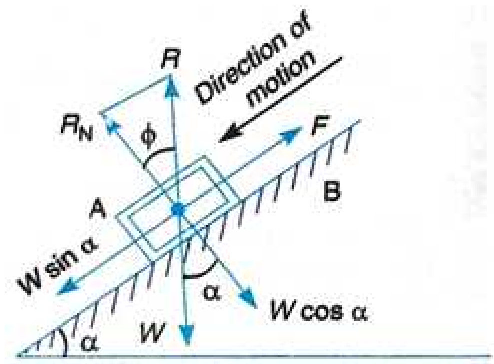
W = weight of body

P = force acting on horizontal direction

N = normal reaction

- when the body just beings to slide the above forces will being equilibrium.
- Limiting angle of friction is the angle (θ) at which the body tends to or just starts to slide.
- The maximum force of friction which acts at this condition is limiting force of friction.

Angle of repose :



$F = w \sin\alpha$

$N = w \cos\alpha$

$F = \mu N$

⇨ $F = \mu W\cos\alpha$

⇨ $W\sin\alpha = \mu W\cos\alpha$

⇨ $\sin\alpha = \mu\cos\alpha$

⇨ $\mu = \frac{\sin\alpha}{\cos\alpha}$

⇨ $\mu = \tan\alpha$

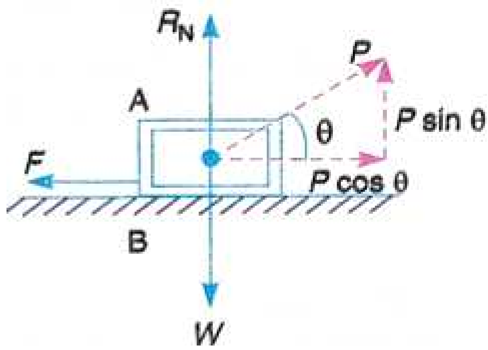
⇨ $\tan\varphi = \tan\alpha$

⇨ $\varphi = \alpha$

- if the body is in equilibrium in inclined plane with angle α then the angle of repose (α)in equal to the angle of friction (θ).
- If the angle of inclination (α) of the plane to the horizontal plane is such that the body beings to move down the plane then the angle (α) is called angle of repose.

Minimum force required to slide on a rough horizontal plane :

Here, P = Force acting on the body
W = Weight of the body
N = Normal reaction.
F = Frictional force



Let the P (force) acting on the body at an angle θ to the horizontal.

At equilibrium condition

$P\cos\theta = F$ -----------------(i)

$N + P\sin\theta = W$ -----------(ii)

⇨ $N = W - P\sin\theta$

$P\cos\theta = \mu \times N$

⇨ $P\cos\theta = \mu(W - P\sin\theta)$

⇨ $P\cos\theta = \tan\varphi\,(W - P\sin\theta)$

⇨ $P\cos\theta =$ [illegible] $-\ P\sin\theta)$

⇨ $P\cos\theta \times P\cos\varphi = W\sin\varphi - P\sin\theta \times \sin\varphi$

⇨ $P(\cos\theta \times \cos\varphi + \sin\theta \times \sin\varphi) = W\sin\varphi$

⇨ $P\cos(\theta - \varphi) = W\sin\varphi$

⇨ $P =$ [illegible]

To minimum value of P then

$\cos(\theta - \varphi) = 1$

⇨ $\cos(\theta - \varphi) = \cos 0$

⇨ $(\theta - \varphi) = 0$

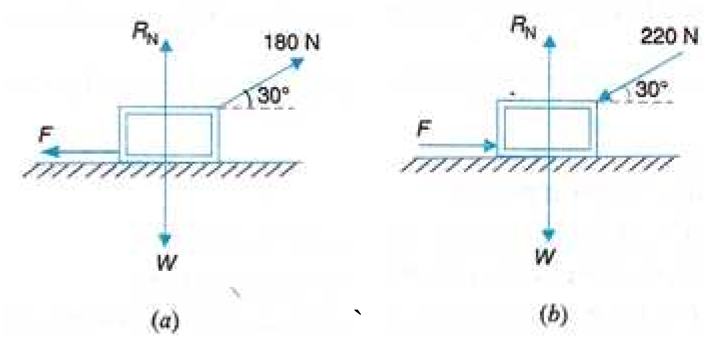
⇨ $\Theta = \varphi$

Due to the maximum value domination then minimum value of numerator or value of P

$$\mathbf{P_{min} = W\sin\varphi}$$

Question-1

A body resting on rough horizontal plane required a pull of 180N inclined at 30° to the horizontal plane just to move, it was found that a push of 220N inclined at 30° to the horizontal plane just move the body. Determine the weight of body and co-efficient of friction.



Answer :

Here,

Case – 1

$F = 180\cos 30°$

$W = N + 180\sin 30°$

$\mu N = 180\cos 30°$

$w = N + 90$

⇨ $N = 155.88/\mu$

⇨ $W = 155/\mu + 90$ ---------------(i)

Case -2

$F = 220\cos 30$

⇨ $F = 190.52$

⇨ $\mu N = 190.52$

$W + 220\sin 30 = N$

⇨ $W = N – 110$

⇨ $N = 190.52/\mu$

⇨ $W = 190.52/\mu – 110$ ------------(ii)

From equation-(i) & equation-(ii)

$$155.88/\mu + 90 = 190.52/\mu - 110$$

⇨ $\frac{155.88 + 90\mu}{\mu} = \frac{190.52 - 110\mu}{\mu}$

⇨ $90\,\mu + 110\,\mu = 190.52 - 155.88$

⇨ $\mu = \frac{34.64}{200}$

⇨ $\mu = 0.1732$

put the value of μ in equation (ii)

$$W = 190.52/\mu - 110$$

⇨ $W = 190.52/0.1732 - 110$

⇨ $W = 990N$

Friction on Rough inclined plane:

Considering the motion of the body up the plane.

W = Weight of the body.

Angle of inclination to the horizontal.

Limiting angle of friction for the contact surface.

P = Effort applied in a given direction in order the slide the body with uniform velocity considering friction.

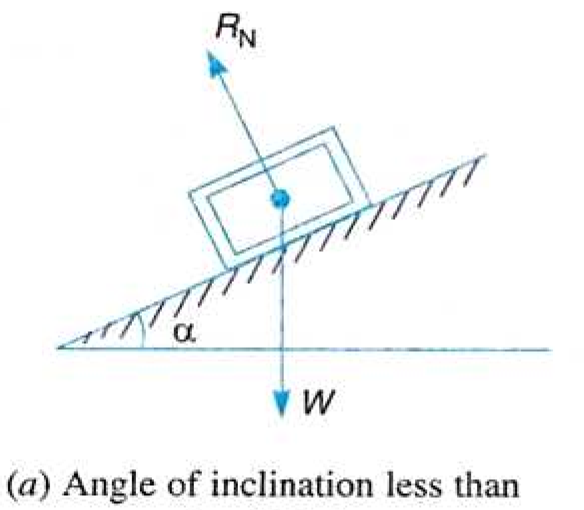
P_0 = Effort applied to moved the body up neglecting friction.

Angle between P and weight of the body W.

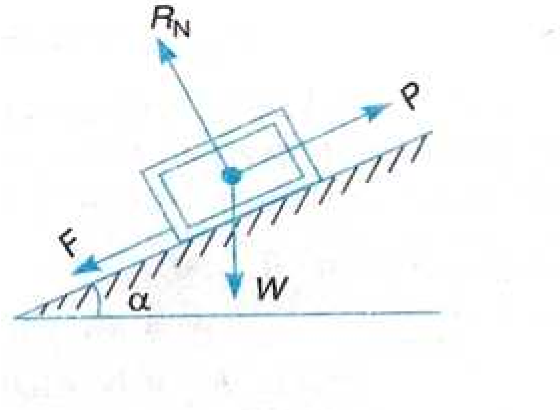
Co-efficient of friction.

N = Normal reaction.

R = Resultant reaction.

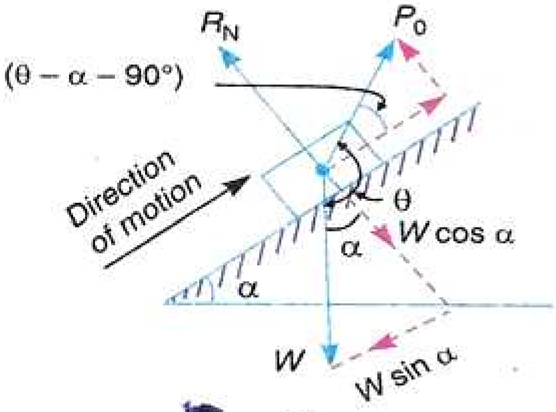


(*a*) Angle of inclination less than angle of friction.



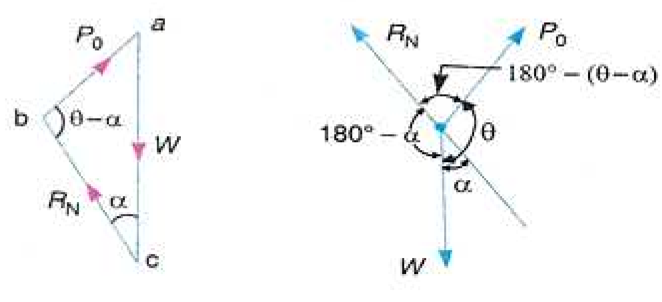
(b) Angle of inclination more than angle of friction.

Without friction:



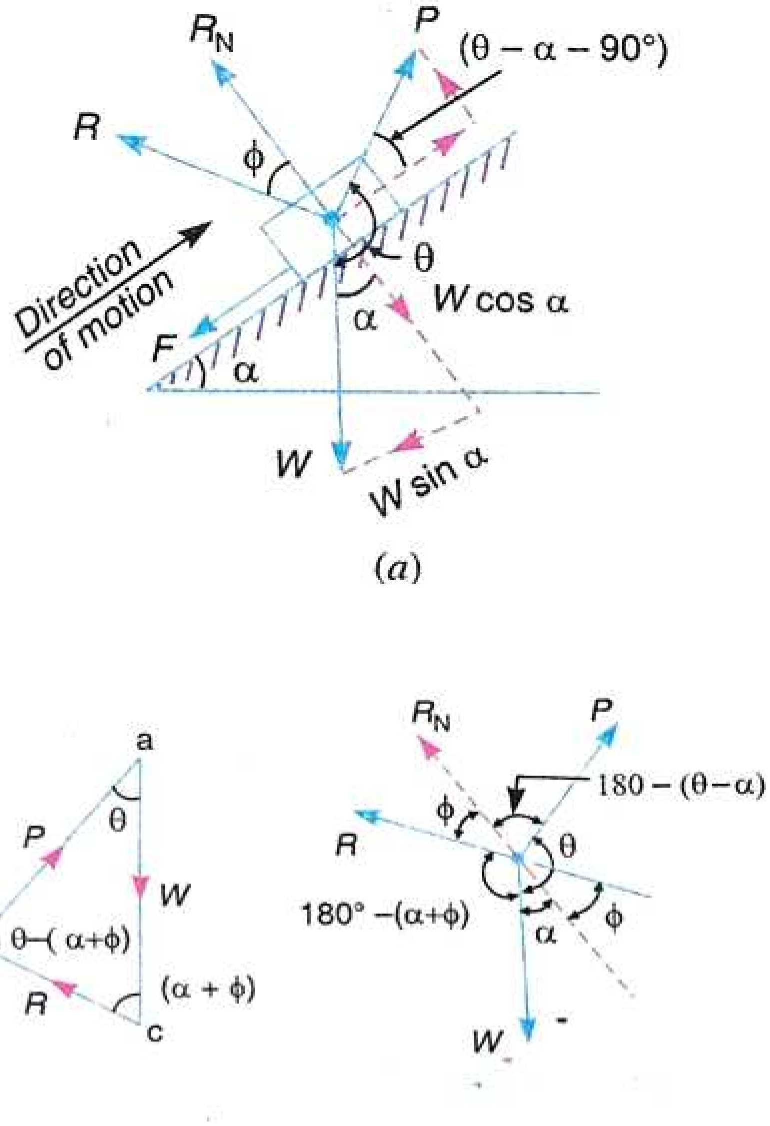
(a)

Applying lami's theorem



(b) (c)

By considering friction:



(a)

(b)

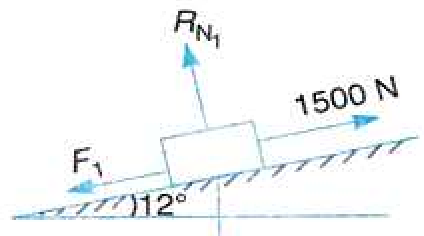
(c)

Numerical problems:

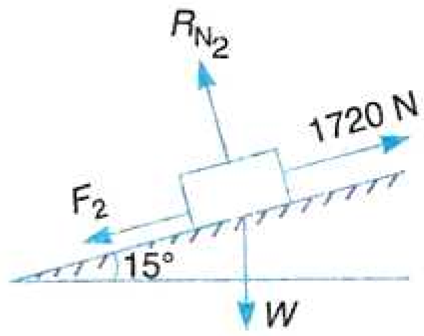
Problem-1

An effort of 1500 N is required to just move a certain body upon inclined plane of angle 12 with the horizontal, force acting parallel to the plane. If the angle of inclination is increased to 15 then the effort required is 1720 N find the weight of the body & co-efficient of friction.

Soplution:



(a)



(b)

W = 7500N.

$\sigma = 0$

$1500 = F_1 + \;2$

$= \;1 + \;2$

$= \mu$ wcos 12+ w sin 12

$\Rightarrow 1500 = w(\sin 12 + \mu \cos 12)$

$\Rightarrow 1500 = w(0.207 + \mu . 0.97)$..........................(1)

$\Rightarrow 1720 = w(\sin 15 + \mu \cos 15)$

$\Rightarrow 1720 = w(0.25 + \mu . 0.96)$..............................(2)

Dividing equation -1 to equation -2, we get

$1720/1500 = w(0.25 + \mu . 0.96)/ w(0.207 + \mu . 0.97)$

$\Rightarrow 1500(0.25 + \mu . 0.96) = 1720(0.207 + \mu . 0.97)$

$\Rightarrow 1440\mu + 388 = 356.4 + 1560.04\mu$

$\Rightarrow 133.38\mu = 29.96$

$\Rightarrow \mu = 29.96/ 133.38 = 0.13112$

So w = 4676.17N

Considering the motion of the body down the plane :-

Neglecting friction:-

Considering friction:-

Efficiency of the inclined plane:-

Let us consider the following two cases:

For the motion of the body up the plane:

$=$

$= \times$

$= \times$

$= \times (-)$

$= \times \sin . \cot(? + Ø) - \cos$

$=$

If $\theta = 90°$

$\Rightarrow ? =$

$=$

If $= 90$

$\Rightarrow ? =$

$=$

$=$

For the motion of the body down the plane:

If = 90°

If = 90 +

SCREW FRICTION:

- The fastening like screws, bolts, nuts etc have screw threads which are made by cutting a continuous helical groove on a cylindrical surface.
- If threads are cut on the outer surface of the solid rod, these are known as outernal threads. & if the threads are cut on the internal surface of a hollow rod then these are known as internal threads.

Helix:

Path traced by a thread while moving along a screw thread.

Pitch:

It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to axis of the screw.

(a) Development of a screw.

Lead:

Distance of a screw thread advanced axially in one turn

Lead = pitch (single thread)

Lead = n x pitch (multi thread)

Depth of thread:

Distance between top & bottom surfaces.

Or distance between crest & root.

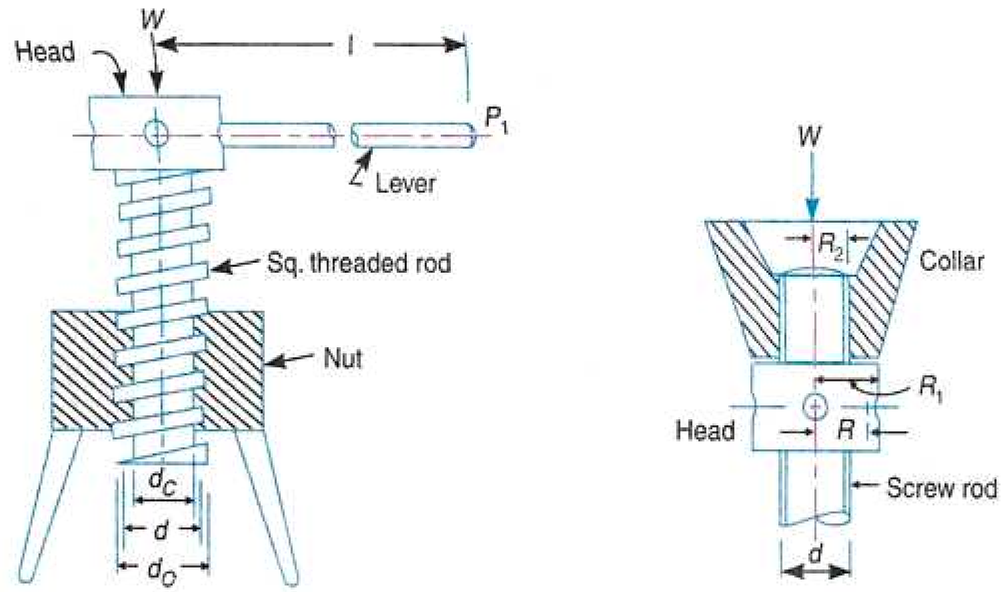
Helix angle(α):-

$$\tan\alpha = \frac{p}{\pi d}$$

Where d = mean diameter.

Friction in screw and nut:- (up)

The motion of a nut on a screw is similar or analogous to the motion on an inclined plane when the nut rises the motion is similar to up the plane taking friction into consideration.



(a) Screw jack.

(b) Thrust collar.

Mechanical advantage = [illegible]

[illegible] .... (1)

From equation-1, [illegible]

If [illegible]

[illegible]

= [illegible]

= cot ([illegible])

M.A = cot ([illegible])

Velocity ratio = [illegible]

[illegible]

V.Rb = cot?

Mechanical efficiency = [illegible]

Down:-

[illegible] ....(1)

From equation-1

M.A = W/P

= [illegible]

[illegible] = 90°

? [illegible] = [illegible]

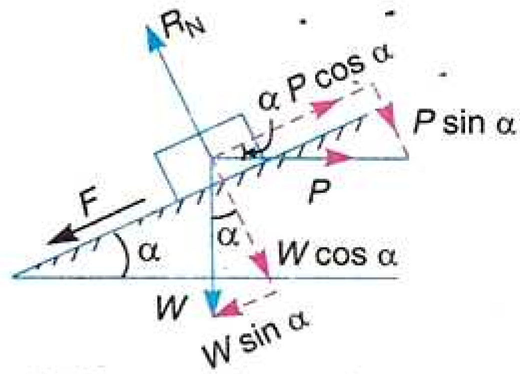
= [illegible]

= cot(? - Ø)

[illegible]

Torque required to lift the load by a screw jack:-

Up:



(*b*) Forces acting on the screw.

Along the plane:

p cos| = w sin| [illegible]

⇒. p cos| = w sin| [illegible] ....(1)

Perpendicular to plane, N= w. cos| [illegible] ....(2)

In place of 'N' put equation -2

P cos| [illegible]

[illegible]

⇒ [illegible]

⇒ [illegible]

⇒ [illegible] = w ([illegible])

⇒ p.cos(| [illegible]) = w sin [illegible]

⇒ p = [illegible]

= tan (? + Ø)

Torque, T = P ×

Where P is the effort applied at the circumference of the screw it lift the load if we rae applied load by leaver then T = P X l

V.R =

M.A =

M.A = cot(? + Ø)

η =

⇒ η =

Power = TW

=

speed in rpm.

W = angular velocity.

Problem-2:

A screw jack has a thread of 10 mm pitch what effort applied at the end of a handle having 400 mm long will be required to lift a load at of 2KN if the efficiency 45 %.

Solution:

given data:

Pitch (Pc) = 10 mm

Long (l) = 400mm

Weight (W) = 2KN

Efficiency = (η) = 45% = 0.45

$\eta = \frac{}{?? Ø?}$

$\Rightarrow 0.45 = \frac{}{?? Ø?}$

$\frac{}{?? Ø?} = 0.45$

$\Rightarrow \tan(? + Ø) = 8.84 \times 10^{?}$

$P = w.\tan(? + Ø)$

$= 2 \times 10^{?} \times 8.84 \times 10^{?}$

= 17.68 N .

? effort applied at the end is 17.68 N.

Problem:

An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 KN at a speed of 300mm/min. The screw has a single square thread of 6mm pitch of a diameter of 37mm. The co-efficient of friction at the screw thread is 0.1 estimate the power of the motor.

Solution:

given data:

Load(w) = 75 KN $= 75 \times 10^{?}$

Speed = 300mm/min

Pitch(Pc) = 6mm

Diameter (d) = 37mm

Co-efficient =0.1(tan)

N = 300/6 = 50 rpm

$P = w.\tan(? + Ø)$

$\tan ? =$ [illegible]

$p =$ [illegible] $= 11.43 \times 10$ [illegible]

$T =$ [illegible] $\times$ [illegible]

$= 11.43 \times 10$ [illegible] $\times$ [illegible]

$\text{Power} = Tw = T \times$ [illegible]

$= 1108$ [illegible]

$= 1.1$ kw

Relation between load and effort when the load lifted up in a simple screw jack:-

Up:

$p = w.\tan?(? + Ø?)$ ………………………….(1)

[illegible] $=$ [illegible] $\times$ [illegible]

$\Rightarrow P =$ [illegible] ……………………….(2)

From equation-1 and equation-2

[illegible] $= w.\tan(? + Ø)$

$\Rightarrow P_1 =$ [illegible] $\tan(? + Ø)$

$\Rightarrow P_1 =$ [illegible]

$=$ [illegible]

M.A =

= w×

=

V.R =

η =

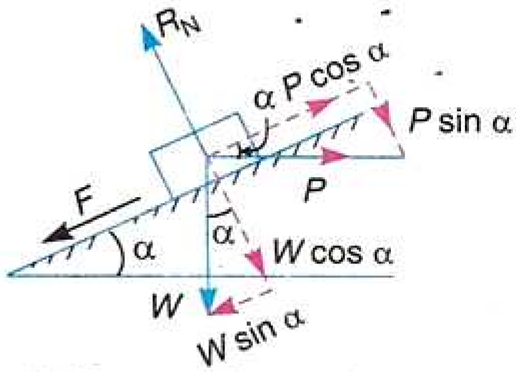
=

=

=

Relation between load & effort when load is morning down in a simple screw jack:-

Down:



(*b*) Forces acting on the screw.

P = W tan([illegible]) ……………………………… (i)

$P_1 l$ = P X [illegible]

⇒ P = [illegible] ……………………………… (ii)

From equation (i) & (ii)

[illegible]

⇒ P_1 = [illegible]

⇒ P_1 = [illegible]

= [illegible]

⇒ P_1 = [illegible]

M.A = [illegible]

Problem:

A screwjack has a square threaded screw of 7.5cm mean diameter. The angle of inclination of the thread is 3 & the [illegible] is 0.6 it is operated by a handle of 45cm long what pull must be exerted at the end of the handle to (i) raise & (ii) lower down a load of 1000N.

Answer:

Given data:

D = 7.5 Cm tan Ø = 0.6

[illegible] $= 3^0$ ⇒ Ø = tan^{-1}(0.6) = 30.96°

$\mu = 0.6$

l = 45 cm

w = 1000 N.

(i) $P_1 =$ [illegible] {tan (α + Ø)}

= [illegible] {tan(3 + 30.96)}

= 561.24

(ii) $P_1 =$ [illegible] {tan (α − Ø)}

= [illegible] {tan(3 - 30.96)}

= (- 44.23)

= 44.23

Problem:

A simple screw jack has a square threaded screw of mean diameter 9cm, the pitch is 10mm μ is 0.12 . it is operated by a handle of 60 cm long to raise & lower down a load of 2000 KN. Find out the efficiency in both the cases.

Answer:

Given data:

D = 9cm =0.9 mm

Pc = 10 mm

μ = 0.12

= 2000 = 2000 × 10

L = 60 cm = 6 mm

P_1 =

= 312724.32

= 312.72 KN

M.A =

=

= 6.39

V.R =

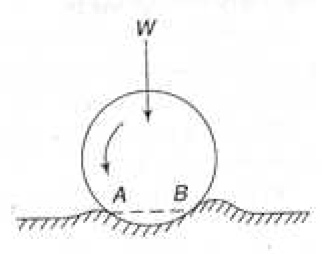
= = 3.76

? = = = 1.7

Rolling friction:

Concept:

When a cylinder rolls over a flat surface, it makes a line contact parallel to the axis of the cylinder, and when a sphere roll, over a flat surface it makes a point contact there is no sliding at the point or line contact there is no sliding at the point or line contact, so the above two are example of pure rolling, but in actual practice, pure rolling never occurs as there is always some resistance to the rolling.



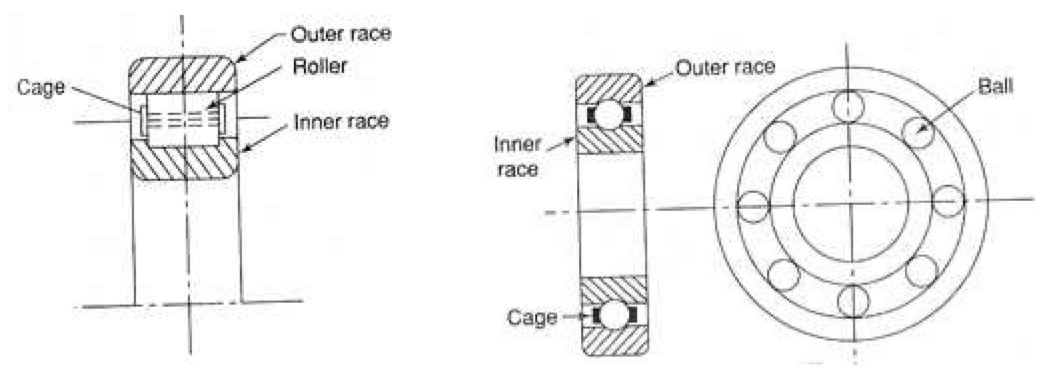
The contact surface of the two bodies (sphere or cylinder and surface) are always more or less deformed by thge redaction between them, so that the ideal point or te line contact degenerates into an area of contact.

The harder the materials in contact, the lesser the deformation of contact surface. Even the hardest materials have a deformation (may be small).

Bearings:

Bearing is a mechanical element that permits relative motion between the two parts, such as the craft and the housing with minimum friction, rolling contact bearings are called anti- friction bearings

Ball bearings:



- Inner race is moving
- Outer race is stationary
- Anti friction element i.e. ball/roller/needle
- Cage which separates the balls or rollers from each other

Function of bearings:

- The bearing ensures free rotation of the shaft or the angle with minimum friction
- The bearing supports the shaft or the axle and holds it in the correct position.
- The bearing takes up the forces that act on the shaft or the axle and transmits them to the frame and foundation.

Applications: (rolling contact bearings)

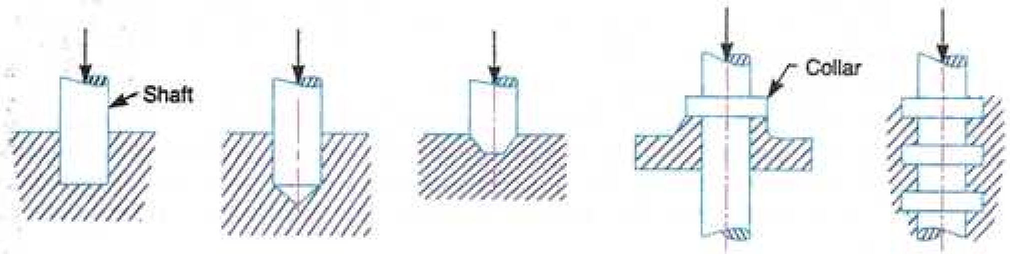
- Machine tool spindles
- Automobile front or rear axles
- Gear boxes
- Small size electric motors
- Rope sleeves, crane hooks and hoisting drums

Applications: (sliding contact bearings)

- Cranck shaft bearing in diesel and petreol engines
- Steam and gas turbines.
- Concrete mixtures
- Rope conveyors

Pivot & collar friction :

The rotating shafts are quite frequently subjected to axial load which is known as thrust. This axial load produces lateral motion of the shaft along its axis which is not desirable. In order to prevent the lateral motion of shaft one or more bearing surfaces called pivots & collars are provided.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

A bearing surface provided at the end of the shaft is known as pivot & a collar is provided along with the length of the shaft with bearing surface of revolution.

Pivot bearing is same foot step bearing.

EX: shaft of steam turbin, propeller shafts of ships.

During motion rudding velocity is V

Rate of ware PV Pr = constant rate of ware

prw pr = c

P = intensity of press on the bearing surface

R = radious of the bearing

= angular velocity of the shaft

Assumption:

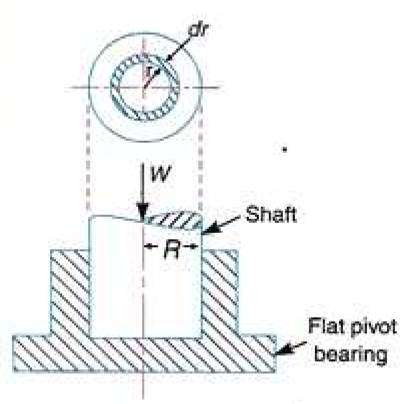
The disof press is uniform.

The wear is uniform.

Pivot bearing:

(i) Flat pivot bearing.
(ii) Conical pivot bearing.

Flat pivot bearing:



Flat pivot or footstep bearing.

By considering uniform pressure

W = load transmitted over the nearing surface.

R = radious of bearing surface

P = intensity of press

Let a small ring at a radious of ‘r’ having its radious or thickness dr.

Area of small bearing surface (

Frictional resistance = ?

= ?

= 2

Fr = ? 2 = ? G

To reque on the ring Tr = Fr x r

= 2 μ ρ r.dr × r

= 2 μ ρ r.dr

T = ? 2 μ ρ r

= 2 μ ρ ?

= 2 μ ρ

= 2 μ ρ R

=

T = (3)

Power lost in friction:

P = Tw (4)

W =

N = speed in rpm.

P =

=

=

P =

Considering uniform wear:

Pr = constant

Pr = c

P = c/r

= × 2

W = ? × 2

= ? × 2

=? 2

=2 ?

C =

Torque (Tr) = 2

⇒T = ? 2

= ? 2

= ? 2

= 2 ?

=

=

T =

Problem:

A vertical shaft 150mm in diameter rotating at 100rpm rests on a flat end footstep bearing the shaft carries a vertical load of 20 N. Assuming estimate the power lost in friction.

Solution:

Given data:

D = 150mm = 0.5m.

R = 75mm

N = 100mm

W = 20n

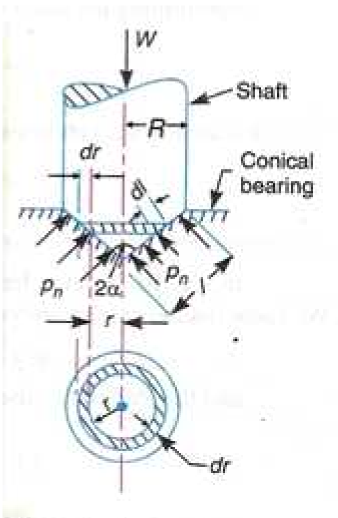
0.05

P =

= ▯G

= 523.54 watt (ans)

Conical pivot bearing:



Conical pivot bearing.

W = load carried by a shaft supported by a conical pivot bearing.

Pn = intensity of press normal to the cone.

α = samiangle of the cone.

μ = co-efficiency of friction between the shaft & the bearing.

R = radious of the shaft.

Consider a small ring of radius ‘r’ & thickness ‘dr’.

Let dl is the length of the ring along the cone.

$\sin\alpha = \frac{dr}{dl}$

$\Rightarrow dl = \frac{dr}{\sin\alpha}$

Dl = dr cosecα (i)

Area of the small ring

$= 2\pi r\, dl$

$= 2\pi r\, dr \operatorname{cosec}\alpha$

Considering uniform pressure:

$= p_n \times 2\pi r\, dr \operatorname{cosec}\alpha$

$= p_n .\sin\alpha$

$= p_n \times 2\pi r\, dr \operatorname{cosec}\alpha . \sin\alpha$

$= p_n \times 2\pi r\, dr$

W= ?

= ? × 2

= × 2 ?

= 2 [ ] R

Friction resistance (Fr) =

= . × 2 sec α

Fr = 2

Frictional torque acting on the ring

Tr = Fr x r

=2 ×

= 2

Total frictional torque. T= ? 2

= 2

= 2 [

= 2 × …………………….. ( )

Substituting the value of ‘p[illegible] in equ (i)

T = 2

= × ×

T =

T= [? =

Considering uniform wear:

p_r = normal intensity of pressure at a distar 'r' from the central axis.

p_r. R = c

⇒ p_r = c/r

Load transmitted to the ring

$\delta W = p_r \times$

$= p_r \times 2$

$= \frac{c}{r} . 2$

$= 2$

Total load transmitted to thge bearing

W= ?

= ? 2

=2 ?

=2 [ ]

= 2

C =

Fr = . ?

= . × ?

Tr = Fr.r

= ?

= 2

Total frictional torque acting on the bearing

T=? 2

=? 2

= 2

= 2 [

T = R^2 -------------------- (i)

Substituting the value of 'c' in equation (i)

T = R^2

= × × R^2

T = ×

oR T = × [? =]

Problem:

A conical pivot supports a vertical shaft of 200mm diameter subjected to a load of 30KN. The cone angle is 120^0 & co-efficient of friction is 0.025 find the power lost in friction when the speed is 140rpm, assuming

1. uniform pressure
2. Uniform wear

Solution:

Given data:

D = 200mm = 30KN

= 120 = 0.025

N = 140rpm.

Uniform pressure:

T=

= 0.025 × 30 × 100 × 120

= 57.73

P = Tw

= T×

= 57.73×

= 846.36 watt

Uniform wear:

T = × ×

= [illegible]

= 43.301

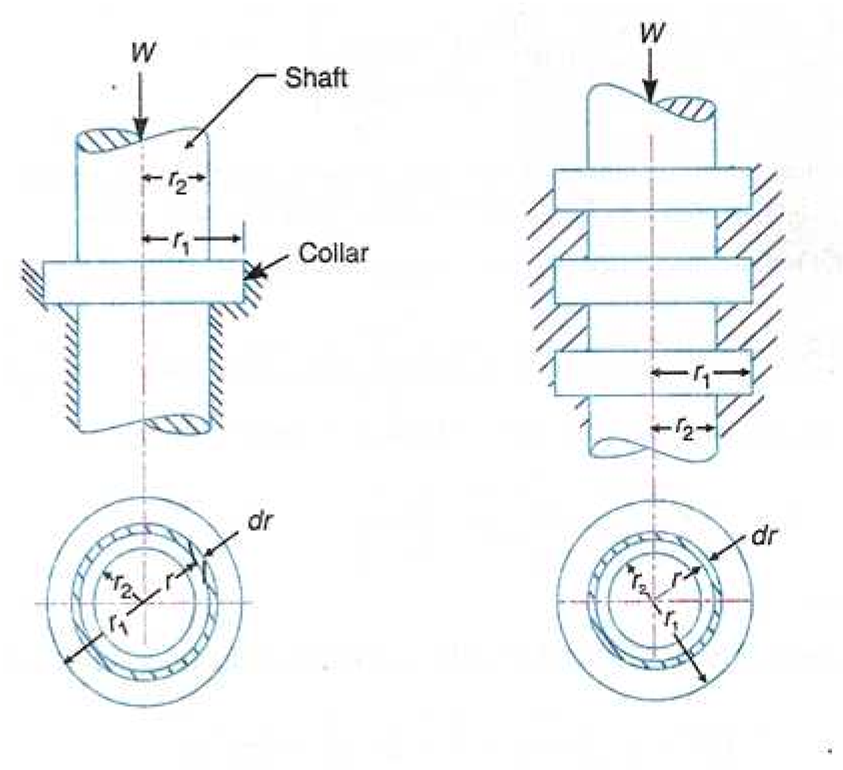
P = Tw

= T [illegible]

= 43.301 [illegible]

= 634.82 watt

Flat collar bearing:



(a) Single collar bearing (b) Multiple collar bearing.

Collar bearing are also known as thrust bearings.

r_1 = external radius of collar

r_2 = internal radius of collar.

Considering uniform pressure:

$P = \frac{W}{A}$

$P = \frac{W}{\pi\{(r_1)^2 - (r_2)^2\}}$

$Tr = Fr \times r$

$= \mu\, \delta W \times r$

$= \mu \times p \times 2\pi r\, dr \times r$

$= \mu \times p \times 2\pi r^2\, dr$

$T = \int_{r_2}^{r_1} \mu \times p \times 2\pi r^2\, dr$

$= 2\pi\mu p \int_{r_2}^{r_1} r^2\, dr$

$= 2\pi\mu p \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$

$= \frac{2}{3}\pi\mu p \{(r_1)^3 - (r_2)^3\}$

$= \frac{2}{3}\pi\mu \times \frac{W}{\pi\{(r_1)^2 - (r_2)^2\}} \times \{(r_1)^3 - (r_2)^3\}$

$T = \frac{2}{3}\mu W \frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}$

Considering uniform wear:

$Pr = \text{constant} = C$

$P_1 r_1 = P_2 r_2 = c$

$\delta W = p \times 2\pi r\, dr$

$= 2\pi C\, dr$

= 2 (? –)

W = ?

= ?

= 2 ?

= 2 (

c =

Fr = .

Tr = F.

= . .

= . 2

=

T = ?

= 2 ?

= 2 [

= 2 × (

T = [

T =

T = [? (

Problem:

A thrust shaft of a ship has 6 collars of 6000mm external diameter & 300mm internal diameter.otal thurst from the propeller is 1000KN if the co-efficient of friction is 0.12 & speed of the friction at the thrust block,assuming uniform pressure, uniform wear.

Ans: (1)P = 26.4KW (ii) P=25.45KW

Let n is the no. Of collar in multicollar

? = [illegible]

? = [illegible]

[illegible]:

N = 6,

$D_1 = 600$,

$d_2 = 300$,

$r_1 = 300mm$,

$r_2 = 150mm$

W = 1000KN

$\mu = 0.12$, = 90

i) Uniform pressure:

T = [illegible]

= [illegible] 0.12 × 1000 × [illegible]

= 28000

$P = TW = 28000 \times$ [illegible]

$= 263893.78$ watt

$= 263.8$ kW

<u>ii) uniform wear:</u>

$T =$ [illegible] [[illegible]]

$=$ [illegible] $0.12 \times 1000 \times (300 + 150)$

$= 27000$

$P = TW = 27000 \times$ [illegible]

$= 254469.$watt

$= 254.4$ Kw

Problem:

A shaft has a no. Of collars in triangle with it the external diameter of the collars is 400mm & the shaft diameter is 250mm if the intensity of pressure is $0.35 n/nm^2$ & the co-efficient of friction is 0.05 estimate.

i. he power observed when the shaft runs at 105rpm carrying a load of 150Kn
ii. No.of collers required

Solution:

Given data:

$D_1 = 400mm$,

$r_1 = 200mm = 0.2m$

[illegible] $= 0.05$

D_2 = 250mm,

r_2 =125mm

N =105rpm

W= 180KN

P=0.35N/mn^2

T= [illegible]

$= [illegible] \times 0.05 \times 150 \times 1000$ [illegible]

= 1240.38N m

P = [illegible] × [illegible]

$= [illegible] \times 1240.38$

= 13638.69w

=13.638 Kw

(ii) No. Of collars

? = [illegible]

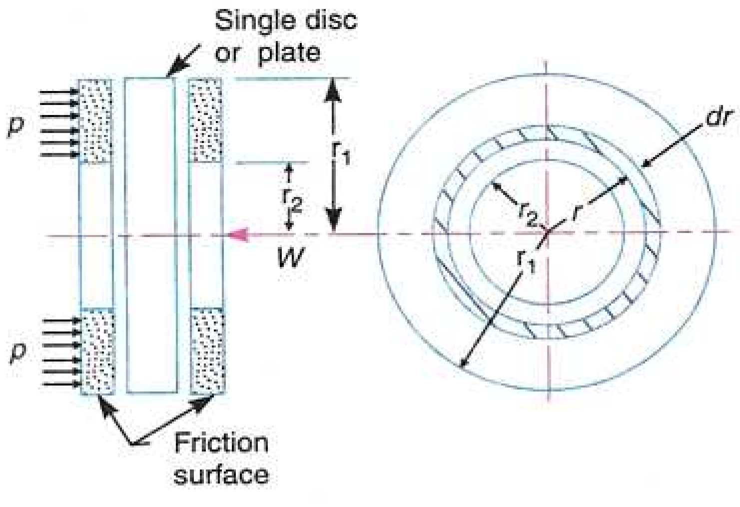
= [illegible]

= 5.59 = 6

<u>Friction clutches:</u>

A friction cloth has it principal application in the transmission of the power shafts and mechanics which must be started and stopped frequency.

<u>Single disc or plate clutch:</u>



(a) (b)

A single disc or plate clutch consists of a clutch plate whose both sides are faced with a friction material.

Considering 2 frictional surfaces, maintained in contact by an axial thrust W.

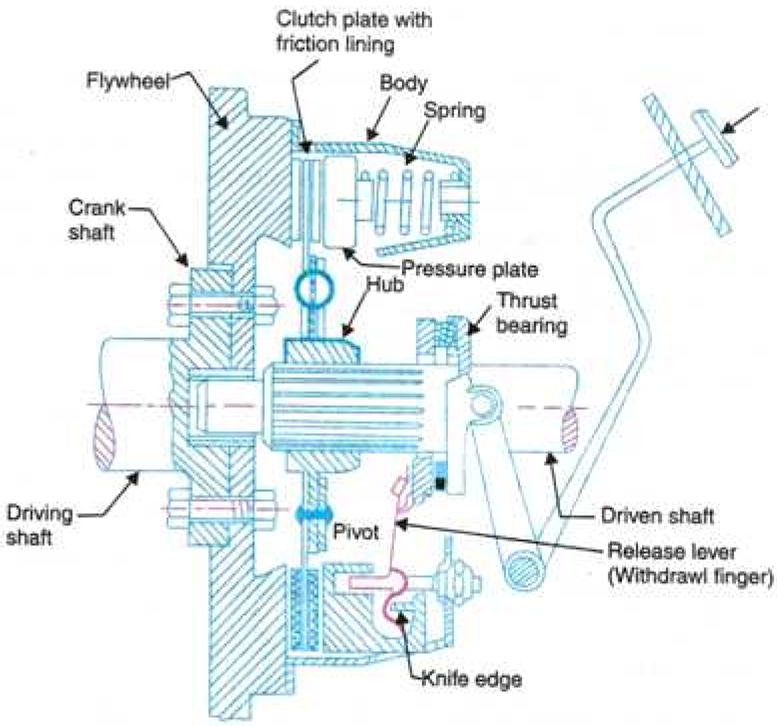
T = torque transmitted by the clutch.

P = intensity of axial pressure with which the contact surface are held together.

R_1 & r_2 = internal and external radius of friction faces.

Co-efficient of friction.

Area of the ring



<u>By considering uniform pressure:</u>

Normal or axial force on the ring

=

Fr =

=

=

Tr = Fr

=

T =

=

=

$= \times \times$

$= \times [$

Considering uniform wear:

Pr = c

$= \times 2$

Fr =

$= \times \times 2$

$= 2$

Tr = Fr.r

$= 2$

T = ?

=

=

T =

T =

In case of two surface torque transmitted

T = n

Where n = 2

$P_{max} \times r_{min} = C$

$\Rightarrow P_{mix} x r_2 = C$

$\Rightarrow P_{mix} =$ [illegible]

$P_{min} X r_{max} = C$

$\Rightarrow P_{min} x r_1 = c$

$\Rightarrow P_{min} =$ [illegible]

$P_{avr} =$ [illegible]

The uniform pressure theory gives a higher frictional torque then the unform wear theory.

Therefore uniform wear should be consider unless otherwise if not started.

Problem:

A single plate clotch, with both sides effective has outer respectively the maxinternal pressure at any point in the contact surface is not to exceed $0.1 N/mm^2$ .if co-effrient of friction is 0.3. betermined the power transmitted by a clutch at a speed of 2500rpm.

Ans: given data:

$D_1 = 300mm$,

$r_1 = 150mm$

$D_2 = 200mm$, $r_2 = 100mm$

$n = 2$,

$P_{max} = 0.1 N/mm^2$

= 0.3

N = 2500rpm.

$P_{max} = c/r_c^2$

$\Rightarrow c = p_{max} \times r_2$

$= 0.1 \times 100$

$= 10$

C =

? = 2 ?

$= 2 \times \quad \times 10(150 - 100)$

= 3141.59N

T = n ( )

$= 2 \times 0.3 \times 3141.59( )$

= 235619.25N-mm

= 235.619N-m

Power = T

$= 235.619 \times \quad = 23$

= 61684.90 watt

= 61.68 KW. Ans

Multiple disc clutch:

$T = \eta$

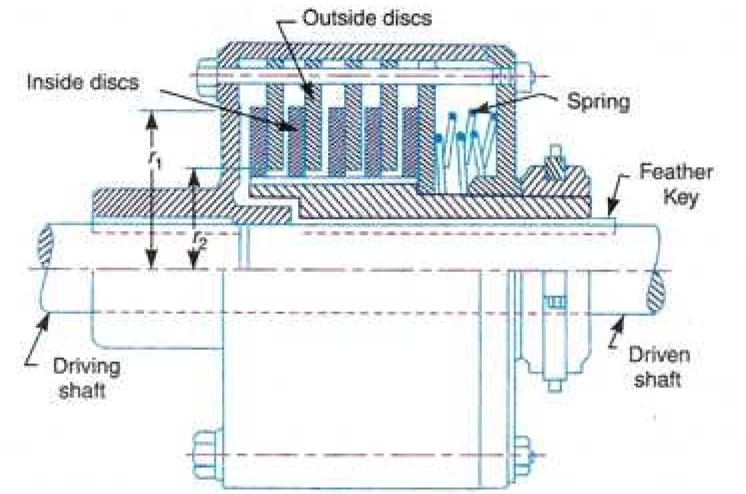
$\eta = (n_1 + n_2) - 1$

in case of multiple disc clutch

η = no. Of pair clutch

n_1 = no. Of disc on the driving shaft

n_2 = no. Of disc on the driven shaft



In a multi plate clutch, the no. Of frictional lining and the metal plates is increased with increases the capacity of a clutch to transmit torque. The above figure shows a simplified diagram of a multi plate clutch.

Thed friction rings are splendid on their outer circumference and engaged with corresponding splines on the flyu wheel. There are free to slide axialy. The friction material thus, rotates with fly wheel and the engine shaft. The no. of friction rings depends upon the torque to be transmitted.

The driven shaft also support disc on the splines which rotates with the driven shaft and can slide axially. If the actuating force on the pedal is removed, a spring presses the disc into contact with the friction rings and the torque is transmitted between the engine shaft and the driven shaft.

If n is the total no. of plates both onn the driving and the driven members, the no. of active surface will be n – 1.

Problem:

A multiplate disc clutch transmits 50kw of power at 1800rpm. Co-efficient of friction for the friction surfaces is 0.1. Axial intensity of presser is not to exceed 160kn/m^2. The internal radius is 80mm & is 0.7times the internal radius. Find the no of plates needed to transmits the required torque.

Solution:

Given data :

Power = 55kw = 55x10^{3}w

N = 1800rpm

= 0.1

Pmar =160KN/m^2

=160x10^{3}N/m^2

R^2 = 80mm = 0.08mt

R^2 = 0.7×

⇒ = 0.114

= 2

$= 2 \times \times 160 \times 10^3 \times 0.08 \times (0.114 - 0.08)$

= 2734.4N

= 2.73 KN

P =Tw

? =

=

Total torque 291.79 N.m.

One surface

T =

= 0.1× 2734.4 ×

= 26.52

η =

= 11

No. Of plate = n + 1 = 12 ans.

Problem:

A single dry plate clutch transmits 75kw at 900rpm . The axial pressur is limited to 0.07N/mm^2 .If co-efficient of friction is 0.25 , find:

1. Mean radious & face width of the friction lining Assuming = 4
2. External & internal radius of the clutch plate.

Solution:

given data:

P = 7.5 Kw = 7.5×10^{3}w

N = 900rpm.

P_{max} = 0.07N/mm^2

μ = 0.25

= 4

$\Rightarrow$ = 4

$\Rightarrow$ = 4

$\Rightarrow$ × = 4

$\Rightarrow$

$\Rightarrow 8$

$\Rightarrow 8$

$\Rightarrow 7$ ……………. ( )

$\Rightarrow 7$

$\Rightarrow$

P = T×

$\Rightarrow$ T = =

= 79.57

Brake:

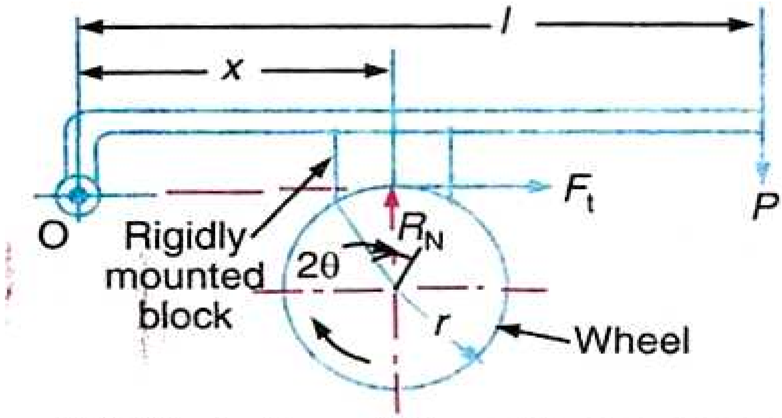
A brake is a device by means of which artificial frictional resistance is applied to a moving m/c member, in order to retard or stop the motion of a m/c.

(Depending upon the arrangement)

Barkes are classified as follows

1. Hybraulic brake
2. Electric brake
3. Mechanical brake

Single block or shoe brake:



(*a*) Clockwise rotation of brake wheel

P = force applied at the end of the lever.

Rn = normal force (pressing the brake block on the wheel).

R = radious of the wheel.

2θ = angle of contact of the black

Ft = tangential breaking force/frictional force.

μ = co-efficient of friction.

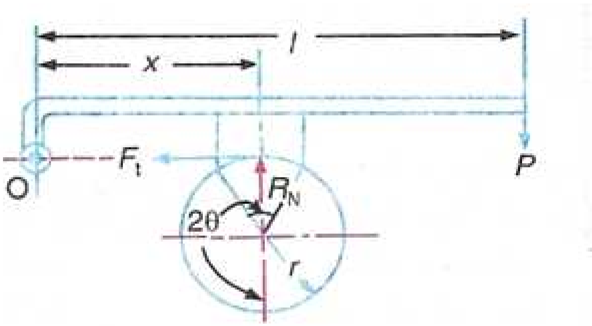
T_B = breaking torque.

Ft =

T_B = Ft

Case-1:

Considering Ft passes through the point 'o' taking moment about the point 'o'.



(*b*) Anticlockwise rotation of brake wheel.

σ = 0

P

R_N =

F_t=

=

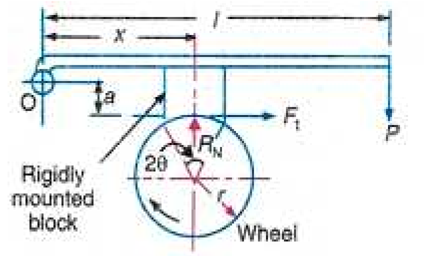
T_B = F_t

=

T_B =

Case-ii:

F_t passes through a distance below the point 'o'.



(a) Clockwise rotation of brake wheel.

σ =0

P

=

= N(

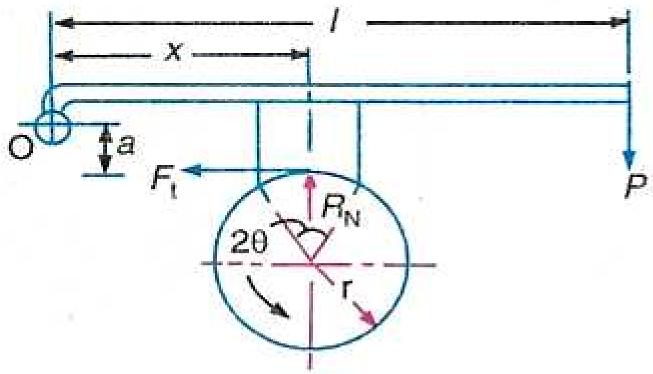
R_N =

F_t = =

$T_B = F_t$

T_B =

Clockwise movement:



(b) Anticlockwise rotation of brake wheel.

Taking the movement about the point' o'

P

=

=

$R_N =$

$F_t =$

=

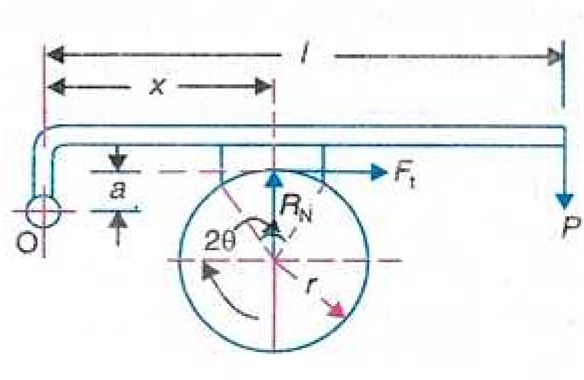
$T_B = F_t$

$T_B =$

Case-3:

F_t passes through a distance of 'a' above 'o'

Clockwise:



(*a*) Clockwise rotation of brake wheel.

Taking moment about the point 'o'

P

=

$=$

$R_N =$

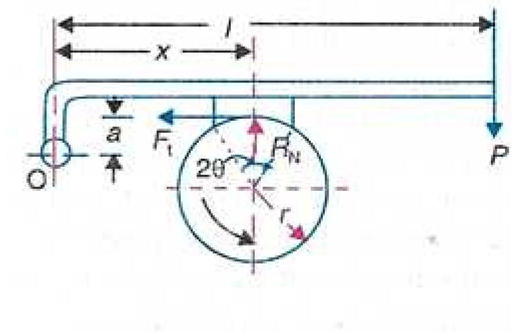
$F_t =$

$=$

$T_B = F_t$

$T_B =$

Anticlock wises:



b) Anticlockwise rotation of brake wheel.

P

$=$

$=$

$R_N =$

$F_t =$

$=$

$T_B = F_t$

Dynamometer:

Dynamometer is device by means of which the energy or work down by a primemovercan be measured.it is a brake having additional device to measure frictional resistance by aling the m/c to run at the rated speed.

Dynamometer:

(i) Absortion dynamometer.

(i) pronybrake dynamometer

(ii) ropebrake dynamometer

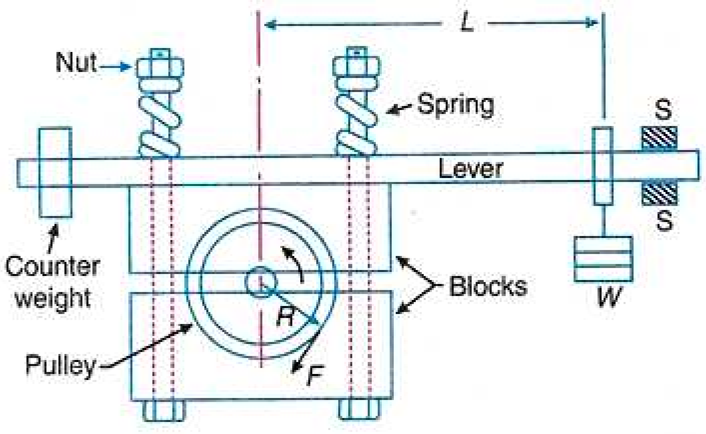
(ii) transmission dynamometer

(i) belt transmission

(ii) epicyclic dynamometer

(iii) torsion dynamometer.

Prong brake dynamometer:



W= weight at the outer end of the lever in N.

L=distance between the centre of the pulley to the weight w.

F = frictional resistance between the blocks & the pulley in N.

R = radius of the pulley in mt.

N = speed of the shaft in R.P.M.

Moment of the frictional resistance or torque on the shaft

T = F.R = W× N.M

Work down/min = T × 2

Power = watt.

Rope Brake Dynamometre

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consist of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a bead weight as shown in figure in order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let: W = dead load in newtons,

S = spring balance reading in newtons,

D = diameter of the wheel in metres,

d = diameter of rope in metres, and

N = speed of the engine shaft in r.p.m.

? Load on the brake

= (W - S)N

We know that distance moved in one revolution

= (D + d)

$\therefore$ Work done per revolution

$$= (W - S)\,\pi\,(D + d) \text{ N-m}$$

and work done per minute

$$= (W - S)\,\pi\,(D + d)\,N \text{ N-m}$$

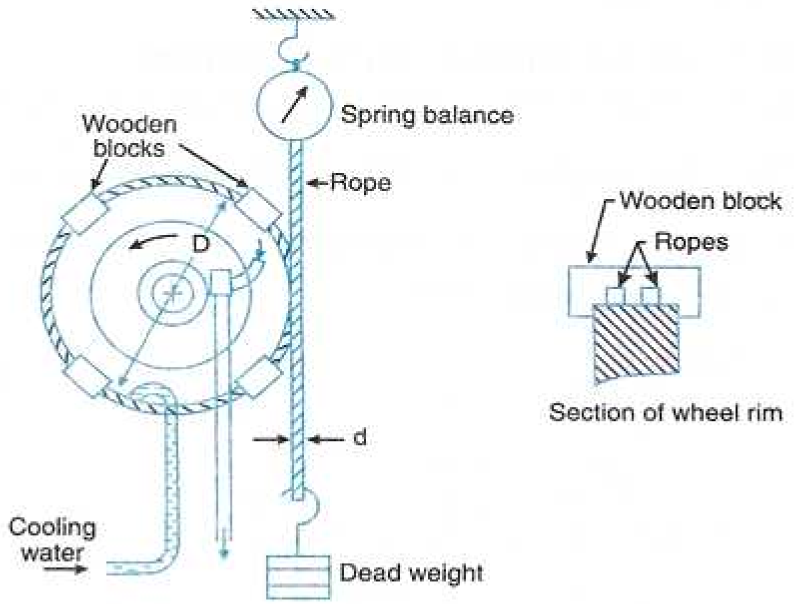


Fig. 19.32. Rope brake dynamometer.

$\therefore$ Brake power of the engine,

$$\text{B.P} = \frac{\text{Work done per min}}{60} = \frac{(W - S)\,\pi\,(D + d) N}{60} \text{ watts}$$

If the diameter of the rope (d) is neglected, then brake power of the engine,

$$\text{B.P.} = \frac{(W - S)\,\pi\,D\,N}{60} \text{ watts}$$

Chapter-3

Power Transmission

Syllabus:

3.1 Concept of power transmission
3.2 Type of drives, belt, gear and chain drive.
3.3 Computation of velocity ratio, length of belts (open & cross) with and without slip.
3.4 Ratio of belt tensions, centrifugal tension and initial tension.
3.5 Power transmitted by the belt.
3.6 V-belts and V-belts pulleys.
3.7 Concept of crowning of pulleys.
3.8 Gear drives and its terminology.
3.9 Gear trains, working principle of simple, compound, reverted and epicyclic gear trains.

Power transmission:-

Depending upon the shapes, cross-section 3 types

1. **Flat belt**
2. **V-belt**
3. **Circular belt**

Depending upon the amount of power transmission

1. **Light**
2. **Medium**
3. **Heavy**

Velocity ratio of open belt drive:-

It is the ratio between the velocities of the driver and the follower or the driven.

Let d_1 = diameter of the driver

d_2 = diameter of the follower

N_1= speed of the driver in r.p.m

N_2 = speed of the follower in r.p.m

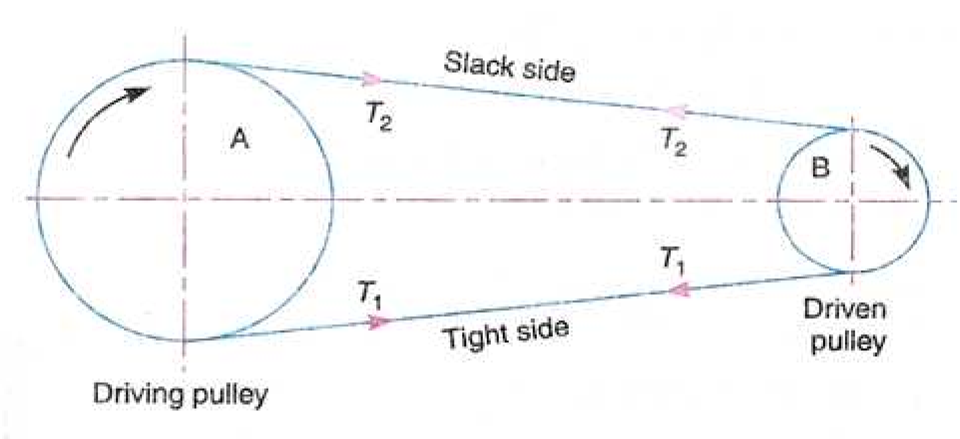
Length of the belt that passes over the driver in 1 minute

= length/min

= 1 1

Length of the belt that passes over the follower in 1 minute

= length/min = 2 2



Since length of the belt passes over the driver in 1 minute = length of the belt passes over the driven in 1 mm.

So (t is not consider)

If it is consider, V.R =

Peripheral velocity of the belt on the driving pulley:-

V_1 = ? V_2 =

If there is no slip, then V1 = V2

Velocity ratio of compound belt drive:-

N2/N1 = D1/D2 , N3/N2 = D2/D3

⇒

=

Slip of belts:-

Sometimes the frictional grips between the belts and shafts becomes in sufficient which may cause some forward motion of the driver without carrying the belt or forward motion of the belt without carrying the driven pulleys or follower pulley with it. This is called slip of belt and is expressed in percentage

Let S_1% = slip between the driver and the belt

S_2%= slip between the belt and the follower

Velocity of belt passing over the driver/sec

$$V = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100}$$

$$V = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Velocity of belt passing over the follower/sec

$$\frac{\pi d_2 N_2}{60} = V - V \times \frac{S_2}{100}$$

$$= V\left(1 - \frac{S_2}{100}\right)$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100} - \frac{S_2}{100} + \frac{S_1 \times S_2}{100 \times 100}\right)$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1 + S_2}{100}\right)$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{S}{100}\right)$$

Where S = S1+S2(total % in slip)

$$\Rightarrow D_2 . N_2 = D_1 . N_1\left(1 - \frac{S}{100}\right)$$

$$? \ \frac{N_2}{N_1} = \frac{D_1}{D_2}\left(1 - \frac{S}{100}\right)$$

"t" consider, $? \ \frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}\left(1 - \frac{S}{100}\right)$

Tension of an open belt drive:-

r1 and r2 = radius of the larger and smaller pulley

x = distance between centres of pulley that is o1 and o2

L = total length of the belt

O2M parallel to EF and O2M perpendicular to O1E

L = arc GJE = EF + arc FKH +HG

= 2× arc GJ +2EF+2 arcFK……………………(1)

In triangle $o_1 o_2 M$

sin? =

=

= ……………… ?2?

1 ? ? ? ……………(3)

Arc FK = 2 ? ? ? …………………(4)

EF = O2M

O2M = ? ? 1 2 (1

EF = O2 M = ? (

= x. ? 1 - ?

= x. ?1 - + ? . ? (binomial expansion)

EF = x – ……………………(5)

L = 2 (Arc GJ + EF+ Arc FK)

= 2 [illegible]

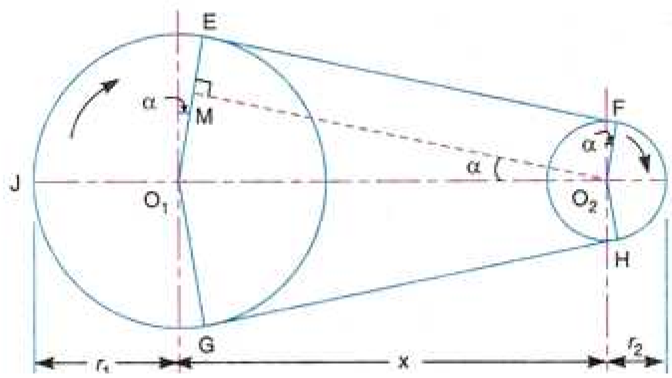
When [illegible] is very very small and as in radian. So, [illegible]

L = 2 [illegible]

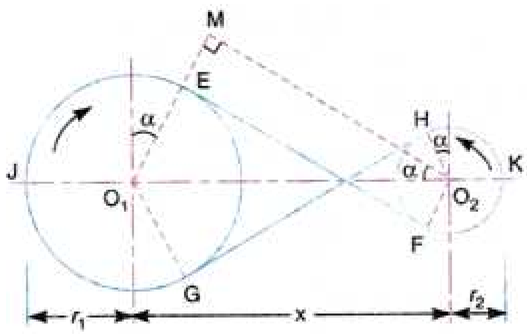
= 2. [illegible]

= 2. [illegible]

So ,L = [illegible]



Cross belt drive:-



L = arc GJE + EF+ arc FKH +HG

= 2.arc GJ +2 EF+2 arc FK

= 2(arc GJ + EF+ arc FK)

In triangle O_1 O_2 M , sin[illegible] ……………(1)

..............(2)

Arc FK = ...(3)

EF = O2M

O2M =

EF = O2 M =

= x.

= x. (binomial expansion)

EF = x –

L = 2. (arc GJ + EF+ arc FK)

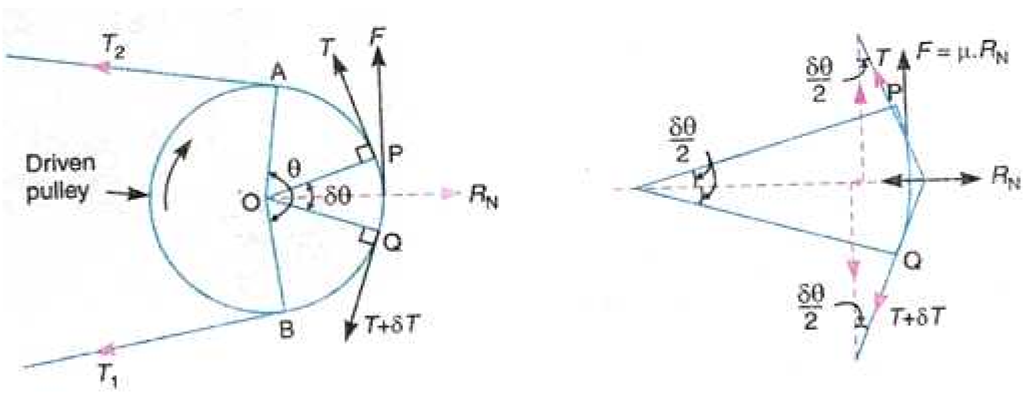
L =

RATIO OF DRIVING TENSION FOR FLAT BELT DRIVE :-

T1 = tension in the belt on the tight side

T2 = tension in the belt on the slack side

θ = angle of contact in radian



Let PQ is in equilibrium under the forces T at P. Tension at Q.($T+\delta T$). Frictional force,$F = \mu R_N$

Where R_N = normal reaction

μ = co-efficient of friction

$\sum F_y = 0$

$R_N = T \sin\frac{\delta\theta}{2} + (T+\delta T)\sin\frac{\delta\theta}{2}$

Since $\delta\theta$ is very small , so $\frac{\delta\theta}{2}$ is very very small

So $R_N = T.\delta\theta$ ……………………..(1)

$\sum F_x = 0$

$F + T.\cos\frac{\delta\theta}{2} = (T+\delta T)\cos\frac{\delta\theta}{2}$

$\Rightarrow F + T = T + \delta T$

$\Rightarrow F = T + \delta T - T$

$\Rightarrow F = \mu R_N = \delta T$

$\mu \times T.\delta\theta = \delta T$

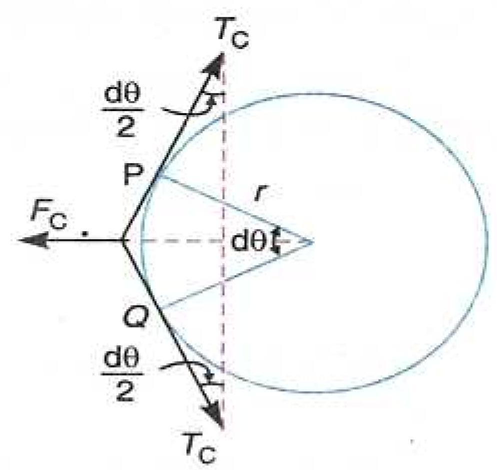
$\Rightarrow \int \frac{dT}{T} = \int \mu\, d\theta$

$\Rightarrow \log_e \frac{T_1}{T_2} = \mu\theta$

$\Rightarrow \frac{T_1}{T_2} = e^{\mu\theta}$

Centrifugal tension:-

Since the belt continuously runs over the pulleys, therefore some centrifugal force is passed whose effect is to increased the tension on both tight and slack sides. The tension caused by centrifugal force is called centrifugal tension.



Considering small portion PQ of the belt having contact angle $d\theta$.

m = mass of the belt per unit length in kg.

v = linear velocity of the belt per unit in metre per second.

R = radius of the pulley over which the belt runs in metre.

T_C = centrifugal tension acting tangentially at P and Q in newton

Length of PQ = r. $d\theta$.

Mass of length PQ = mr $d\theta$

Centrifugal force acting on PQ = F_c = mr $d\theta \times \frac{v^2}{r}$

$$F_c = m\, d\theta\, v^2 \text{ ……………….(1)}$$

σ =0

$T_C.\sin\frac{d\theta}{2} + T_C.\sin\frac{d\theta}{2} = F_c$

As $\frac{d\theta}{2}$ is very small . so $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$

$T_C.\sin\frac{d\theta}{2} + T_C.\sin\frac{d\theta}{2} = F_c$

$\Rightarrow 2.\, T_C \frac{d\theta}{2} = m\, d\theta\, v^2$

$\Rightarrow T_C = m.v^2$

We know from ratio of tension that , 2.3 log

⇒ 2.3 log

Maximum tension in the belt:-

1. Tc is neglected. Maximum tension T_1 = bt

Where = maximum safe stress in newton per metre^2

b= width of belt in mm

t= thickness of the belt in mm

2. Tc is consider

bt +

Power transmission by the belt:-

1. Tc neglected

P = (V watt

2. Tc consider

P =?( ) - ( )?

P = (V watt.

Or , P= (V watt

Initial tension in the belt (To):-

When the pulleys are stationary, the belt is subjected to some tension as the ends are joined together to make the belt continuously moved over the pulleys . the tension is called initial tension.

To= initial tension in the belt

T1 = tension in the tight side of the belt

T2 = tension in the slack side of the belt

? = co-efficient of increase or decrease of the belt length per unit force.

Increase of tension in the tight side = T_1-T_0

Increase of length of the belt in the tight side = ? ([illegible]

Similarly decrease in tension in the slack side = [illegible]

Decrease in the length of the belt on the slack side = ? ([illegible]

Assuming the belt is perfectly elastic such that the length of the belt remain constant. Therefore increase in length on the tight side is equal to decrease in length on the slack side.

Mathematically , ? ? [illegible]

$\Rightarrow T_1 - T_0 =$ [illegible]

$\Rightarrow 2\ T_0 = T_1 + T_2$

$\Rightarrow T_0 =$ [illegible] if Tc is neglected.

If Tc is consider, $T_O =$ [illegible]

Width of belt:-

- At moderate speed the tension in a belt = T_1
- At high speed , taking the centrifugal tension into account, the maximum tension in a belt = Tmax
- To avoid excessive bending stress and to ensure adequate life, the thickness of belt is fixed.
- For soft materials = [illegible] 25
- For normal belt = [illegible] 35
- For stiff belt = [illegible] 50

Problem:

In a flat belt drive the initial tension is 2000N. Co-efficient of friction between the belt and pulley is 0.3 and the angle of lap on the smaller pulley is 150. The smaller pulley has radius of 200 mm and rotates at 500 r.p.m. find the power in KW transmitted by the belt.

Answer:

Given data

To = 2000N

= 0.3

= 150° = 2.61 radian

00mm

d_2 = 400mm= 0.4m

N_2 = 500 r.p.m

V= = 10.47m/s

$T_0 = (T_1+T_2)/2$

2000 = (T_1+T_2)/2

T_1+T_2 = 4000..................(1)

2.3 log

log = 0.3404

=1.4

⇒ $T_1 = 1.4T_2$

T_1+T_2 = 4000

1.4 T_2+ T_2 = 4000

$\therefore 2.4\, T_2 = 4000$

$\Rightarrow T_2 = 4000/2.4 = 1666.66$

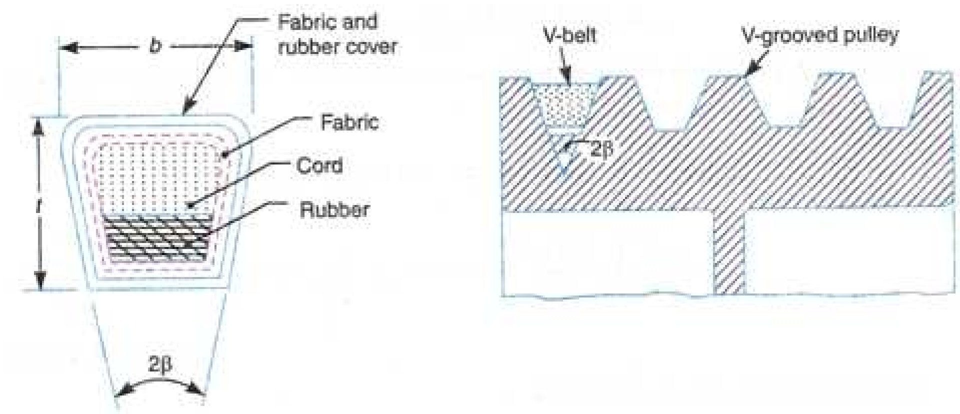
$\Rightarrow T_1 = 1.4\, T_2$

$= 1.4 \times 1666.66 = 2333.33$

V-belts and V-belts pulleys:

We have already discussed that a V-belt is mostly used in factories and workshops where a great amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other.

The v-belts are made of frabic and cords moulded in rubber and covered with fabric and rubber, as shown in figure. These belts are moulded to a trapezoidal shape and aremade endless. These are particularly suitable for short drives when the shafts are at a short distance apart. The included angle for the V-belt is usually from 30^0-40^0. In case of falt belt drive, the belt runs over the pulleys whereas in case of V- belt drive, the rim of the pulley is grooved in which the V-belt runs. The effect of the groove is to increase the frictional grip of the V-belt on the pulley and thus to reduce the tendency of slipping. In order to have a good grip on the pulley, the V-belt is in cotact with the side faces of the groove and not at the bottom. The power is transmitted by the wedging action between the belt and the V-groove in the pulley.

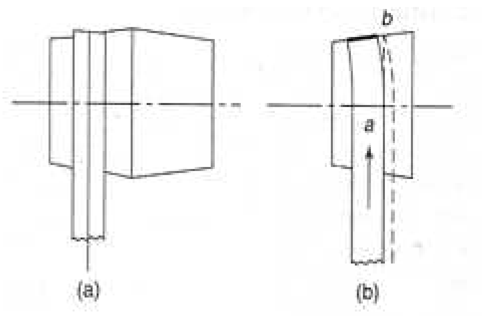


(a) Cross-section of a V-belt. (b) Cross-section of a V-grooved pulley.

A clearance must be provided at the bottom of the groove, as shown in figure in order to prevent touching to the bottom as it becomes narrower from wear. The V-belt drive, may be inclined at any angle with tight side either at top or bottom. In order to increase the power output, several V-belt may be operated side by side. It may be noted that in multiple V-belt drive, all the belts should stretch at the same rate so that the load is equally divided between them. When one of the set of belts break, the entire set should be replaced at the same time. If only one belt is replaced, the new unworm and unstressed belt will be more tightly stetched and will move difference velocity.

3.7 Concept of crowning of pulleys.

As mentioned is section 9.2, the rim of the pulley of a flat-belt drive is slightly crowned to prevent the slipping off belt from the pulley. The crowing can be in the from of conical surface or a convex surface.



Assume that somehow a belt comes over the conical portion of the pulley and takes the position as shown in figure, its centre line remains in a plane, the belt will touch the rim surface at its one edge only. This is impractical. Owing to the pull, the belt always tends to stick to the rim surface. The belt also has a lateral stiffness. Thus, a belt has to bend in the way shown in figure.

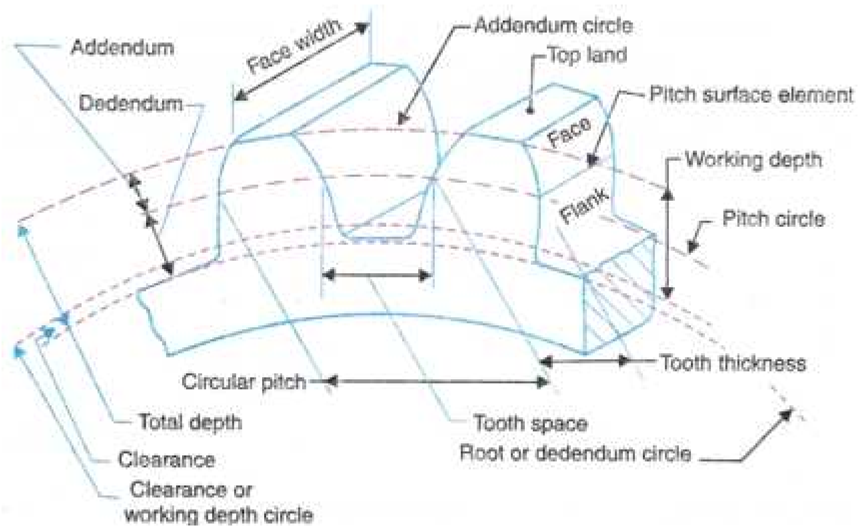
Let the belt travel in the direction of the arrow. As the belt touches the cone, the point a on it tends to adhere to the cone surface due to pull on the belt. This means as the pulley will take a quarter turn, the point a on the belt will be carried to b which is towards the mid-plane of the pulley than that previously occupied by the edge of the belt. But again, the belt cannot be stable on the pulley in the upright position and has to bend to stick to the cone surface, it will occupy the position shown by dotted lines.

Thus, if a pulley is made up of two equal cones or of convex syrface, the belt will tend to climd on the slopes and will thus, run with its centre line on the mid-plane of the pulley.

The amount of crowing is usually 1/96 of the pulley face width.

Terms used in gears:

the folloing terms, which will be mostly used in this chapter, should be clearly understood at the stage. These terms are illustrated in figure.



1. **Pitch circle:**

   It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

   A straight line may also be defined as a wheel of infinite radius.

2. **Pitch circle diameter:**

   It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as **pitch diameter.**

3. **Pitch point:**
   It is common point of contact between two pitch circles.
4. **Pitch surface:**
   It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.
5. **Pressure angle or angle of obliquity:**
   It is the angle between the common normal to two gear teethg at the point of contact and the common tangent at the pitch point. It is usually denoted by Ø.the standard pressure angles are 14 and 20^0.
6. **Addendum:**
   It is the radial distance of a tooth from the pitch circle to the top of the tooth.
7. **Dedendum:**
   It is the radial distance of the tooth from the pitch circle to the bottom of the teeth.
8. **Addendum circle:**
   it is the circle drawn through the top of the teeth and id concentric with the pitch circle.
9. **Dependum circle:**
   it is the circle drawn through the bottom of the teeth. It is also called not circle.

Circular pitch:-

It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is denoted as Pc.

Pc =

d = diameter of the pitch circle

T = no. Of teeth on the gear.

When the two gears will be in mesh Pc should be same for both.

Pc = [illegible]

[illegible]

DIAMETRAL PITCH:-

It is the ratio of no. Of teeth to the pitch circle diameter in mm. It is denoted by Pd. Pd = T/D = [illegible]

Module:-

It is the ratio of pitch circle diameter in mm to the no. Of teeth. It is denoted as “m”.

M = [illegible]

Question:

The no. Of teeth on each of two equal gear in mesh are 40. If the module is 6 mm. Find pitch circle diameter, circular pitch, diametral pitch.

Ans:- given data:- T = 40

m= 6 m

Pd= [illegible]

$\Rightarrow$ Pd = 1/6

$\Rightarrow$ Pd = T/D

$\Rightarrow$ [illegible] = [illegible]

? D = 240 mm

Pc = [illegible] = [illegible] = 18.84

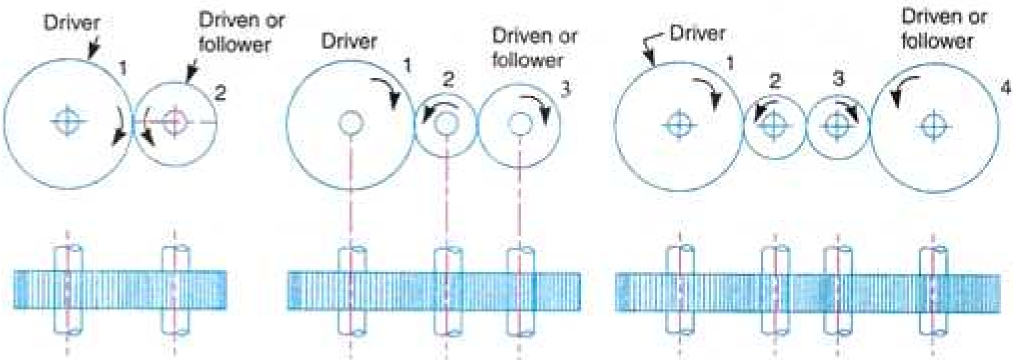
Gear train:-

When two or more gear are made to mesh with each other to transmit power from one shaft to another , such a combination is called gear train.

Different types of gear train:-

1. simple gear train

When there is only one gear on each shaft, as shown in figure. It is known as **simple gear train.** The gears are represented by their pitch circles.



When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in figure. Since the gear 1 drives the gear 2. Therefore gear 1 is called the **driver** and the gear 2 is called the **driven** or **follower.** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

Let: N_1 = speed of gear 1 (or driver) in r.p.m.,

N_2= speed of gear 2 (or driven or follower) in r.p.m.,

T_1= number of teeth on gear 1, and

T_2= number of teeth on gear 2.

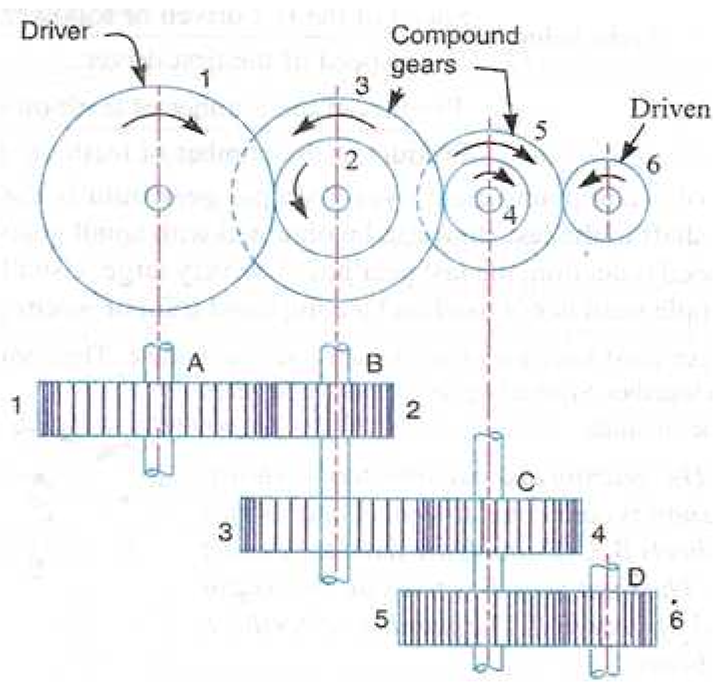
Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speed of any pair of gears in mesh is the inverse of their number of teeth, therefore

Speed ratio =

2. Compound gear train

When there are more then one gear on a shaft, as shown in figure. It is called a **compound train of gear.**

We have seen in art. That the ideal gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.



But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in figure.

In a compound train of gears, as shown in figure, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let: N_1 = speed of driving gear 1,

T_1= number of teeth on driving gear 1,

$N_2, N_3, \ldots\ldots, N_6$ = speed of respective gears in r.p.m.,

and $T_2, T_3 \ldots\ldots, T_6$ = number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

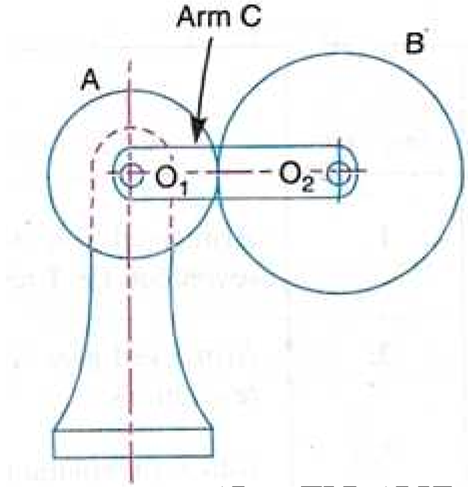
Similarly, for gears 3 and 4, speed ratio is

And for gears 5 and 6, speed ratio is

The speed ratio of compound gear train is obtained by multiplying the equations **(1), (ii) and (iii),**

3. Epicyclic gear train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in figure, where a gear A and the arm C have common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2, about which the gear B can rotate.



We know that speed ratio =

Also ...(N_B = N_c, being on the same shaft)

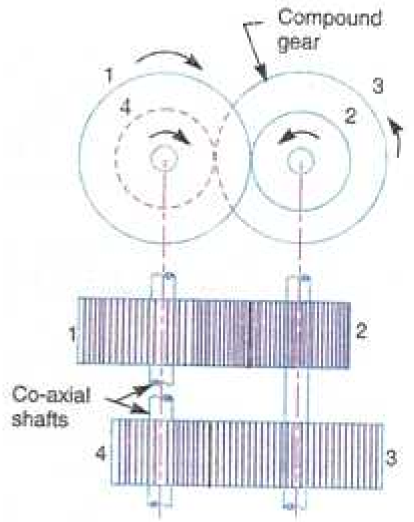
For and to be same, each speed ratio should be so that

Arm is fixed. The gear train is simple and gear A can drive gear B or **vice-versa,** but if gear A is fixed and the arm is rotated about the axis of gear A(i.e. O_1), then the gear B is forced to rotate **upon** and **around** gear A. Such a motion is called **epicyclic** and the gear train arranged in such a manner that one or more of their member move upon and around another member is known as **epicyclic gear trains** (**epi.** Means upon and **cyclic** means around). The epicyclic gear trains may be **simple** or **compound.**

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. Differential gears trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

4. Reverted gear train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as **reverted gear train** as shown in figure.



We see that gear 1 (i.e. first driver) drives the gear 2(i.e. first driven or follower) in the opposite direction, since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3(which is now the second driver) drives the gear 4(i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is like.

Let: T_1= number of teeth on gear 1,

r_1= pitch circle radius of gear 1, and

N_1= speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = number of teeth on respective gears,

r_2 , r_3 , r_4 = pitch circle radii of respective gears, and

N_2, N_3, N_4 = speed of respective gears in r.p.m.

Since the distance between the centre of the shafts of gears 1and 2 as well as gear 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4$$

Also, thye circular pitch or module of all the gear is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

? $T_1 + T_2 = T_3 + T_4$

And speed ratio =

Or

Chapter-4

Governors and Flywheel

Syllabus:

4.1 Function of governor
4.2 Classification of governor
4.3 Working of Watt, Porter, Proel and Hartnel governors.
4.4 Conceptual explanation of sensitivity, stability and isochronisms.
4.5 Function of flywheel.
4.6 Comparison between flywheel & governor.
4.7 Fluctuation of energy and coefficient of fluctuation of speed.

4.1 Function of governor:

The function of a governer is to regulate the main speed of an engine whem there are variations in the load e.g. when the load on an engine increases, its speed decreases therefore it becomes necessary to increases the supply working fluid.

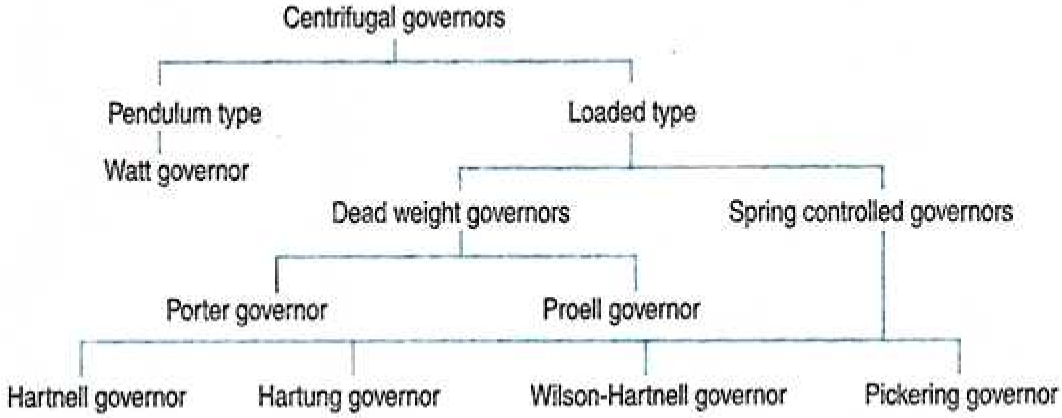
In the other hand, when the load on the engine decreases its speed increases and thus less working fluid is required. The governer automaticallycontrol the supply of working fluid top the engine with the varying load conditions and keep the mean speed within certain limits.

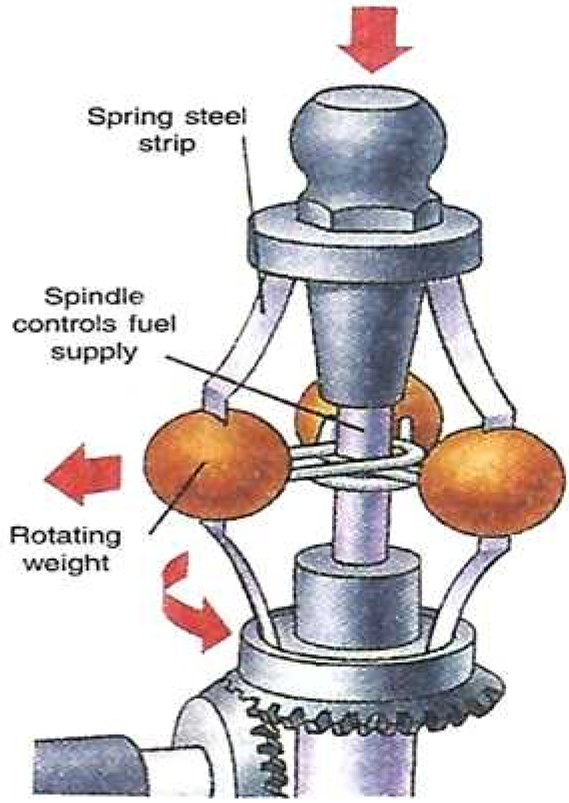
4.2 Classification of governor:

The governors may be classified as

1. **Centrifugal governor**
2. **Inertia governor**

The centrifugal governors, may further be classified as follows :





4.3 Working of Watt, Porter, Proell and Hartnell governors.

Watt governors:

The simplest form of a centrifugal governor is a watt governor, as shown in fig it is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways:

1. The pivot P, may be on the spindle axis as shown in fig.
2. The pivot P, may be offset from the spindle axis and the arms when produced intersect at O, as shown in fig.
3. The pivot P, may be offset, but the arms cross the axis at O, as shown in figure.

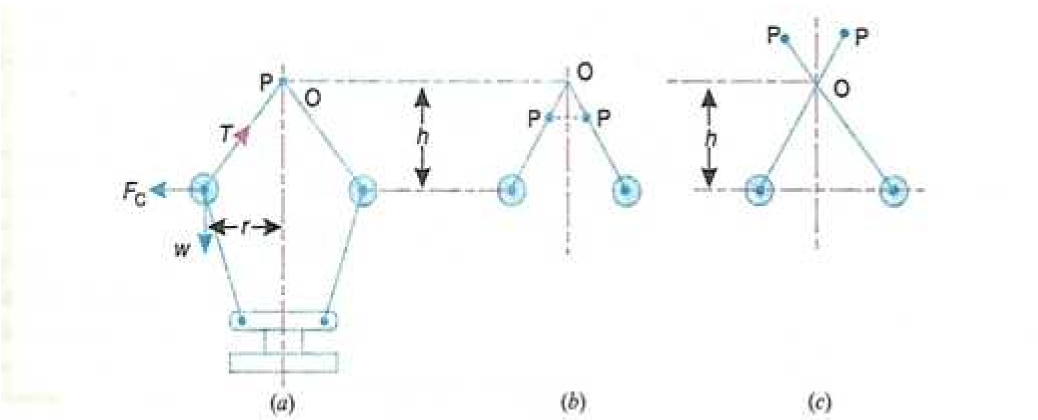


Fig. 1 . Watt governor.

Let

m = Mass of the ball in kg,

w = weight of the ball in newtons = m.g,

T = tension in the arm in newtons,

= angular velocity of the arm and ball about the spindle axis in rad/s,

r = radius of the path of rotation of the ball i.e. horizontal distance from the centre of the ball to the spindle axis in metres,

Fc = centrifugal force acting on the ball in newtons = m. ^{2}.r, and

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of

1. the centrifugal force (F_C) acting on the ball, 2. the tension (T) in the arm, and 3. the weight (w) of the ball.

Taking moments about point O, we have

$$F_C \times h = w \times r = m.g.r$$

or $$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g / \omega^2 \qquad \ldots (i)$$

When g is expressed in m/s^2 and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

$$\omega = 2\pi N/60$$

$$\therefore \quad h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres} \qquad \ldots (\because g = 9.81 \text{ m/s}^2) \ldots (ii)$$

Example *Calculate the vertical height of a Watt governor when it rotates at 60 r.p.m Also find the change in vertical height when its speed increases to 61 r.p.m.*

Solution. Given : $N_1 = 60$ r.p.m. ; $N_2 = 61$ r.p.m.

Initial height

We know that initial height,

$$h_1 = \frac{895}{(N_1)^2} = \frac{895}{(60)^2} = 0.248 \text{ m}$$

Change in vertical height

We know that final height,

$$h_2 = \frac{895}{(N_2)^2} = \frac{895}{(61)^2} = 0.24 \text{ m}$$

$\therefore$ Change in vertical height

$$= h_1 - h_2 = 0.248 - 0.24 = 0.008 \text{ m} = 8 \text{ mm} \textbf{ Ans.}$$

Porter governor:

The porter governor is a modification of a watt's governor. With central load attached to the sleeve as shown in figure the load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Consider the forces acting on one-half of the governor as shown in Fig. (b).

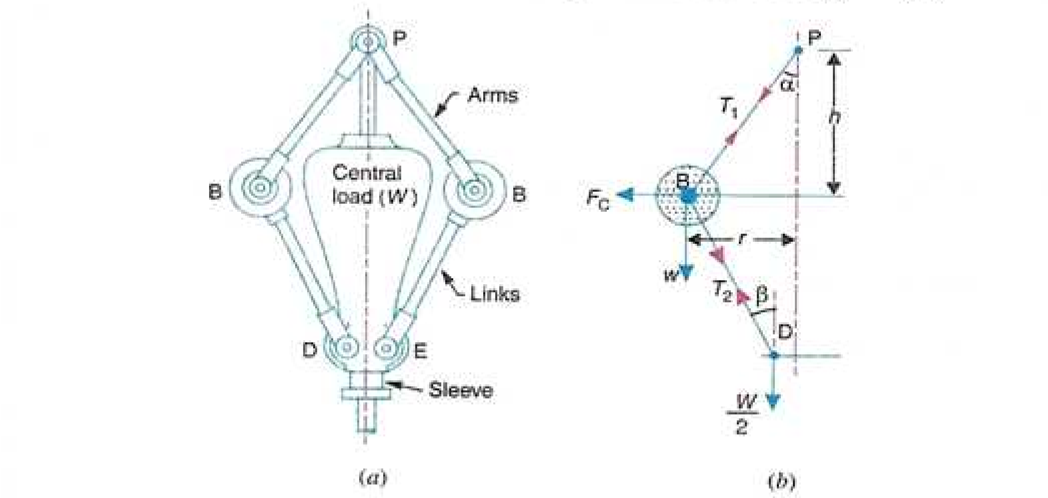


Fig. Porter governor.

Let

m = mass of each ball in kg,

w = weight of each ball in newtons = m.g,

M = mass of the central load in kg,

W = weight of the central load in newtons = M.g,

R = radius of rotation in metres,

Dividing equation (*iii*) by equation (*ii*),

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

or $$\left(\frac{M.g}{2} + m.g\right)\tan\alpha = F_C - \frac{M.g}{2} \times \tan\beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

Substituting $\frac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \frac{r}{h}$, we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q \qquad \ldots (\because F_C = m.\omega^2.r)$$

or $$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$\therefore$ $$h = \left[m.g + \frac{M.g}{2}(1+q)\right]\frac{1}{m.\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2} \qquad \ldots (iv)$$

or $$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q)\right]\frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

or $$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$\therefore$ $$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h} \qquad \ldots (v)$$

$\ldots$ (Taking $g = 9.81$ m/s^2)

2. Instantaneous centre method

In this method, equilibrium of the forces acting on the link BD are considered. The instantaneous centre I lies at the point of intersection of PB produced and a line through D perpendicular to the spindle axis, as shown in Fig. 18.4. Taking moments about the point I,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2} \times \frac{ID}{BM}$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= m.g \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)$$

$$\ldots\left(\because \frac{IM}{BM} = \tan\alpha, \text{ and } \frac{MD}{BM} = \tan\beta\right)$$

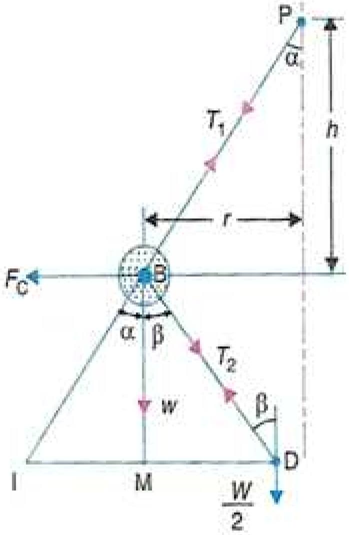


Fig. 18.4. Instantaneous centre method.

Dividing throughout by $\tan\alpha$,

$$\frac{F_C}{\tan\alpha} = m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) = m.g + \frac{M.g}{2}(1+q) \qquad \ldots\left(\because q = \frac{\tan\beta}{\tan\alpha}\right)$$

We know that $F_C = m.\omega^2.r$, and $\tan\alpha = \frac{r}{h}$

$$\therefore \quad m.\omega^2.r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$$

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

... (Same as before)

When $\tan\alpha = \tan\beta$ or $q = 1$, then

$$h = \frac{m+M}{m} \times \frac{g}{\omega^2}$$

Example:

A porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of 5 kg and the mass of the central load on the sleeve is 15 kg. the radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.

Solution:

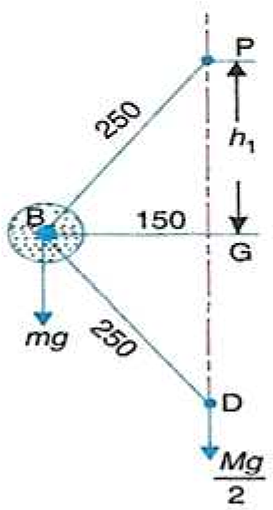
Given:

BP = BD = 250mm = 0.25m

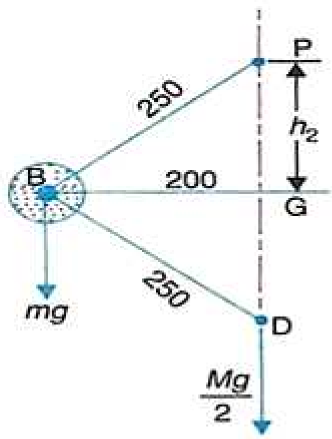
Mass m = 15kg

r_1 = 150mm = 0.15m

r_2 = 200mm = 0.2m



(a) Minimum position.



(b) Maximum position.

The minimum and maximum positions of the governor are shown in Fig. 18.5 (*a*) and (*b*) respectively.

Minimum speed when $r_1 = BG = 0.15$ m

Let N_1 = Minimum speed.

From Fig. 18.5 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.15)^2} = 0.2 \text{ m}$$

We know that

$$(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{5 + 15}{5} \times \frac{895}{0.2} = 17\,900$$

$\therefore$ $N_1 = 133.8$ r.p.m. **Ans.**

Maximum speed when $r_2 = BG = 0.2$ m

Let N_2 = Maximum speed.

From Fig. 18.5 (*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.2)^2} = 0.15 \text{ m}$$

We know that

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+15}{5} \times \frac{895}{0.15} = 23\,867$$

$\therefore$ $N_2 = 154.5$ r.p.m. **Ans.**

Proell governors

The proell governer has the balls fixed at B and C to the extension of the links DF and EG, as shown in figure. The terms FP and GQ are pivoted at P and Q respectively.

Consider the equilibrium of the forces on one-half of the governor as shown in figure. The instantaneous centre (I) lies on the intersection of the line PF produced and the line from D drawn perpendicular to the spindle axis. The perpendicular BM is drawn on ID.

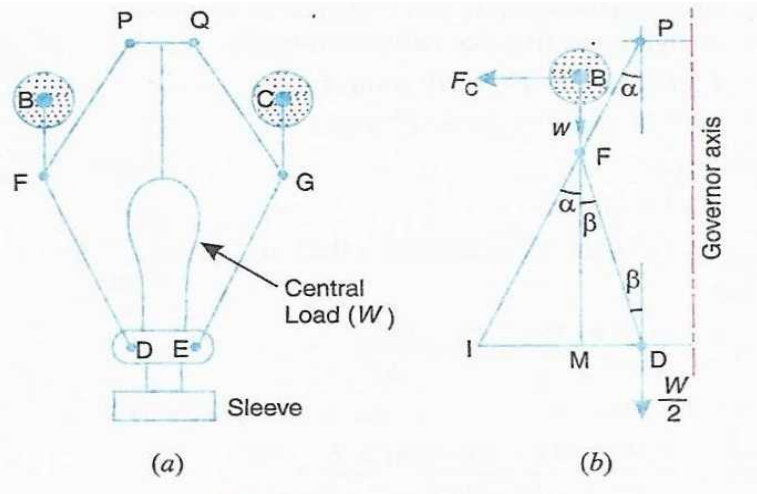


Fig. Proell governor.

Taking moments about I, using the same notations as discussed in Art. 18.6 (Porter governor),

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \qquad \ldots (i)$$

$$\therefore \qquad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \qquad \ldots (\because \; ID = IM + MD)$$

Multiplying and dividing by FM, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

$$\therefore \quad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h}$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81\ m/s^2$, we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h}$$

Example:

A proell governor has equal arms of length 300 mm. the upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. the mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

Solution:

Given:

PF = DF = 300 mm;

BF = 80 mm ;

r_1= 150 mm;

r_2 = 200 mm;

M = 10 kg ;

m = 100 kg;

First of all, let us find the minimum and maximum speed of governor. The minimum and maximum position of the governor is shown in figure.

Let:

N_1= minimum speed when radius of rotation, r_1 = FG = 150 mm; and

N_2 = maximum speed when radius of rotation, r_2 = FG = 200mm.

From figure, we find the height of the governor

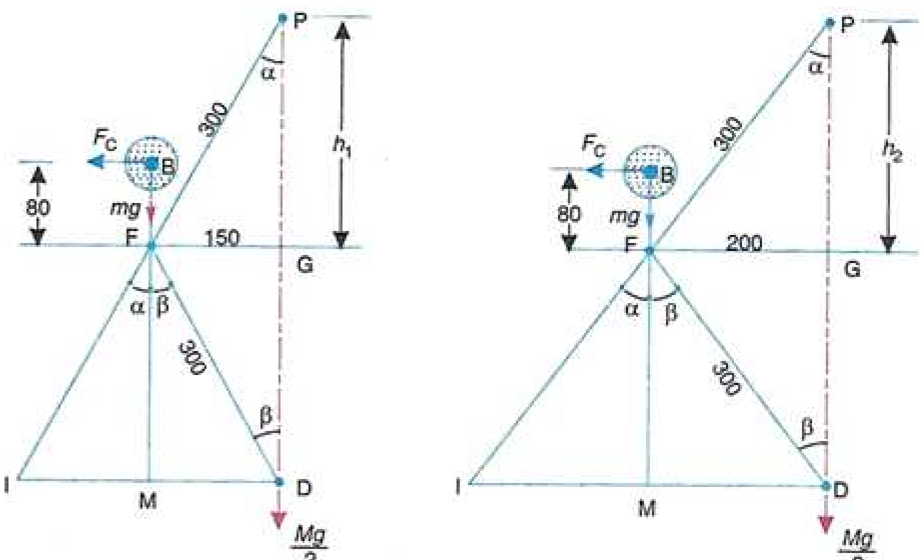
h_1 = PG =

And FM = GD = PG = 260mm = 0.26 m

∴ BM = BF + FM = 80+260

= 340mm = 0.34m

We known that $(N_1)^2$=



All dimensions in mm.

(a) Minimum position. (a) Maximum position.

$$h_2 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm} = 0.224 \text{ m}$$

and $FM = GD = PG = 224 \text{ mm} = 0.224 \text{ m}$

$\therefore$ $BM = BF + FM = 80 + 224 = 304 \text{ mm} = 0.304 \text{ m}$

We know that $(N_2)^2 = \dfrac{FM}{BM}\left(\dfrac{m+M}{m}\right)\dfrac{895}{h_2}$ $\quad \ldots (\because \alpha = \beta \text{ or } q = 1)$

$$= \frac{0.224}{0.304}\left(\frac{10+100}{10}\right)\frac{895}{0.224} = 32\,385 \quad \text{or} \quad N_2 = 180 \text{ r.p.m.}$$

We know that range of speed

$$= N_2 - N_1 = 180 - 170 = 10 \text{ r.p.m.} \textbf{ Ans.}$$

Note : The example may also be solved as discussed below :

From Fig. 18.13 (*a*), we find that

$$\sin\alpha = \sin\beta = 150 / 300 = 0.5 \quad \text{or} \quad \alpha = \beta = 30°$$

and $MD = FG = 150 \text{ mm} = 0.15 \text{ m}$

$FM = FD\cos\beta = 300\cos 30° = 260 \text{ mm} = 0.26 \text{ m}$

$IM = FM\tan\alpha = 0.26\tan 30° = 0.15 \text{ m}$

$BM = BF + FM = 80 + 260 = 340 \text{ mm} = 0.34 \text{ m}$

$ID = IM + MD = 0.15 + 0.15 = 0.3 \text{ m}$

We know that centrifugal force,

$$F_C = m\,(\omega_1)^2\, r_1 = 10\left(\frac{2\pi N_1}{60}\right)^2 0.15 = 0.0165\,(N_1)^2$$

Now taking moments about point *I*,

$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID$$

$$0.0165\,(N_1)^2\, 0.34 = 10 \times 9.81 \times 0.15 + \frac{100 \times 9.81}{2} \times 0.3$$

$$0.0056\,(N_1)^2 = 14.715 + 147.15 = 161.865$$

$$\therefore \quad (N_1)^2 = \frac{161.865}{0.0056} = 28\,904 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$

Similarly N_2 may be calculated.

Hartnell governors

A Hartnell governor is a spring loaded governor as shown in figure. It consists of two bell crank levers pivoted at the points O,O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring force may be adjusted by screwing a nut up or down on the sleeve.

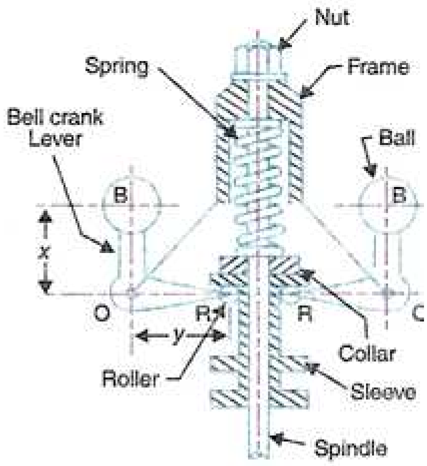


Fig. Hartnell governor.

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres,

r_2 = Maximum radius of rotation in metres,

ω_1 = Angular speed of the governor at minimum radius in rad/s,

ω_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve at ω_1 in newtons,

S_2 = Spring force exerted on the sleeve at ω_2 in newtons,

F_{C1} = Centrifugal force at ω_1 in newtons = $m(\omega_1)^2 r_1$,

F_{C2} = Centrifugal force at ω_2 in newtons = $m(\omega_2)^2 r_2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres,

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

Consider the forces acting at one bell crank lever. The minimum and maximum position is shown in Fig. 18.19. Let h be the compression of the spring when the radius of rotation changes from r_1 to r_2.

For the minimum position *i.e.* when the radius of rotation changes from r to r_1, as shown in Fig. 18.19 (*a*), the compression of the spring or the lift of sleeve h_1 is given by

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \qquad \ldots (i)$$

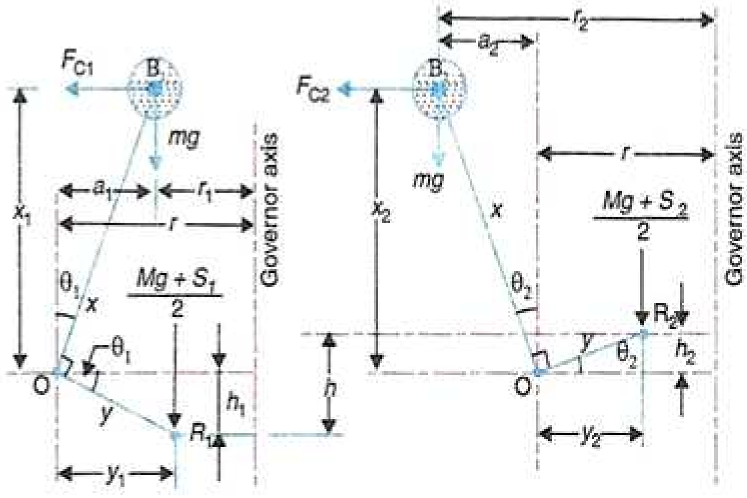
Similarly, for the maximum position *i.e.* when the radius of rotation changes from r to r_2, as shown in Fig. 18.19 (*b*), the compression of the spring or lift of sleeve h_2 is given by

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \qquad \ldots (ii)$$

Adding equations (*i*) and (*ii*),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x} \qquad \ldots (\because h = h_1 + h_2)$$

$$\therefore \quad h = (r_2 - r_1)\frac{y}{x} \qquad \ldots (iii)$$



(*a*) Minimum position. (*b*) Maximum position.

Now for minimum position, taking moments about point O, we get

$$\frac{M.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

or
$$M.g + S_1 = \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1) \quad \ldots (iv)$$

Again for maximum position, taking moments about point O, we get

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

or
$$M.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \quad \ldots (v)$$

Subtracting equation (*iv*) from equation (*v*),

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1)$$

We know that

$$S_2 - S_1 = h.s, \qquad \text{and} \qquad h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \qquad s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms (*i.e.* $x_1 = x_2 = x$, and $y_1 = y_2 = y$) and the moment due to weight of the balls (*i.e.* $m.g$), we have for minimum position,

$$\frac{M.g + S_1}{2} \times y = F_{C1} \times x \quad \text{or} \quad M.g + S_1 = 2F_{C1} \times \frac{x}{y} \quad \ldots (vi)$$

Similarly for maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad M.g + S_2 = 2F_{C2} \times \frac{x}{y} \quad \ldots (vii)$$

Subtracting equation (*vi*) from equation (*vii*),

$$S_2 - S_1 = 2(F_{C2} - F_{C1})\frac{x}{y} \quad \ldots (viii)$$

We know that

$$S_2 - S_1 = h.s, \qquad \text{and} \qquad h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \qquad s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \quad \ldots (ix)$$

Example:

A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. the sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are provided at 120 mm from the governor axis and mass of each ball is 2.5 kg. the ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine :1. Loads on the spring at the lowest and the highest equilibrium speeds, and 2. Stiffness of the spring.

Solution:

Given:

$N_1 = 290$ r.p.m.

or $\omega_1 = 2\pi \times \frac{290}{60} = 30.4$ rad/s;

$N_2 = 310$ r.p.m.

or $\omega_2 = 2\pi \times \frac{310}{60} = 32.5$ rad/s;

$h = 15$ mm $= 0.015$ m;

$y = 80$ mm $= 0.08$ m;

$x = 120$ mm $= 0.12$ m;

$r = 120$ mm $= 0.12$ m;

$m = 2.5$ kg

1. loads on the spring at the lowest and highest equilibrium speeds

Let:

S_1 = spring load at lowest equilibrium speed, and

S_2 = spring load at highest equilibrium speed.

Since the ball arms are parallel to governor axis at the lowest equilibrium speed(i.e. at N_1 = 290 r.p.m.), as shown in figure, therefore

$r = r_1 = 120$ mm $= 0.12$ m

We know that centrifugal force at the minimum speed,

$$F_{C1} = m\,(\omega_1)^2\, r_1 = 2.5\,(30.4)^2\ 0.12 = 277 \text{ N}$$

Now let us find the radius of rotation at the highest equilibrium speed, *i.e.* at $N_2 = 310$ r.p.m. The position of ball arm and sleeve arm at the highest equilibrium speed is shown in Fig. 18.20 (*b*).

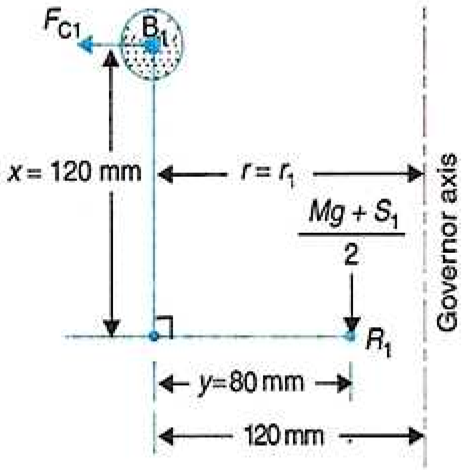
Let r_2 = Radius of rotation at $N_2 = 310$ r.p.m.

We know that $h = (r_2 - r_1)\dfrac{y}{x}$

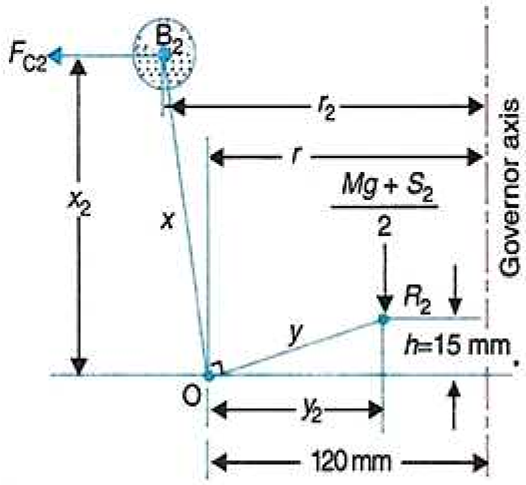
or $$r_2 = r_1 + h\left(\frac{x}{y}\right) = 0.12 + 0.015\left(\frac{0.12}{0.08}\right) = 0.1425 \text{ m}$$

$\therefore$ Centrifugal force at the maximum speed,

$$F_{C2} = m\,(\omega_2)^2\, r_2 = 2.5 \times (32.5)^2 \times 0.1425 = 376 \text{ N}$$



(*a*) Lowest position.



(*b*) Highest position.

Neglecting the obliquity effect of arms and the moment due to the weight of the balls, we have for lowest position,

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 277 \times \frac{0.12}{0.08} = 831 \text{ N}$$

$\therefore$ $S_1 = 831$ N **Ans.** ...($\because M = 0$)

and for highest position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 376 \times \frac{0.12}{0.08} = 1128 \text{ N}$$

$\therefore$ $S_2 = 1128$ N **Ans.** ...($\because M = 0$)

2. *Stiffness of the spring*

We know that stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8 \text{ N/mm} \textbf{ Ans.}$$

4.4 Conceptual explanation of sensitivity, stability and isochronism.

Sensitiveness of governors:

Consider two governor A and B running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor A is greater then the lift of the sleeve of governor B. it is then said that the governor A is more sensitive than the governor B.

In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is sensitiveness of the governor. It may also be stated in another way that for a given lift of the sleeve, the sensitiveness of the governor increase as the speed range decreases. This definition of sensitiveness may be quite satisfactory when the governor is considered as an independent mechanism.

But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a friction as possible of the mean equilibrium speed. The actual displacement of the sleeve is immaterial, provided that it is sufficient to change the energy supplied to the engine by the required amount. For this reason, the sensitiveness is defined as the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

Let N_1 = minimum equilibrium speed

N_2 = maximum equilibrium speed, and

N = mean equilibrium speed

= [illegible]

? [illegible]

= [illegible] = [illegible]

= [illegible] ....(in terms of angular speeds)

Stability of governors:

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e. there is only one radius of the governor, if the equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor ball must also increases.

Isochronous Governors

A governor is said to be isochronous when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronous is the stage of infinite sensitivity.

Let us consider the case of a porter governor running at speeds N1, N_2 r.p.m. we have that

.... (i)

.... (ii)

For isochronisms, range of speed should be zero i.e. $N_2 - N_1 = 0$ or $N_2 = N_1$. Therefore from equations (i) & (ii), $h_1 = h_2$, which is impossible in case of a porter governor. Hence a porter governor cannot be isochronous.

4.5 Function of flywheel.

Fly wheel:-

A fly wheel is used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and relief during the period when the requirement of energy is more than supply.

By supplying the required energy, a fly wheel controls the speed variations caused by the cyclic fluctuation of the engine turning moment.

4.6 Comparison between flywheel & governor.

Governor	Fly Wheel
Its function is to regulate the supply of driving fluid producing energy, according to the load requirements so that at different loads almost a constant speed is maintained.	Its function is to store available mechanical energy when it is in excess of the load requirement and a part with the same when the available energy is less than that required by the load
It is provide on prime movers such as engines and turbines.	It is provided on engine and fabricating machines viz, rolling mills , punching machines shear machines, presses, etc.
It takes care of fluctuation of speed due top variation of load over long range of working of engines and turbines.	In engines it takes care of fluctuations of speed during thermodynamic cycle.
It works intermittantely i.e. only when ther is change in the load	It works continuously from cycle to cycle.
But for governor, there would have been uncessarily more consumpstion of driving fluid. Thus it economises its consumptions	In fabrication machines, it is very economical to use it as its use reduces capital investment on prime movers and their running expenses

Co-efficient of fluctuation of speed:-

The difference between the maximum and the minimum speed during a cycle is called maximum fluctuation of speed.

The ratio of maximum fluctuation of speed to the mean speed is the co-efficient of fluctuation of speed.

Cs =

where N_1 = Maximum speed in r.p.m during the cycle

N_2 = minimum speed in r.p.m during the cycle

N = mean speed in r.p.m =

Cs = [illegible]

Cs = 2. [illegible]

Cs = 2. [illegible]

Cs = 2. [illegible]

4.7 Fluctuation of energy and coefficient of fluctuation of speed.

Fluctuation of energy:

1. The fluctuation of energy may be determined by the turning moment diagram for one complete cycle rotation
2. The vibration of energy above and below the mean resisting torque line in the turning moment diagram are called fluctuation of energy
3. The difference between the maximum and the minimum energy is known as maximum fluctuation of energy.

Maximum fluctuation of energy:

A - G = mean torque line

Let a_1, a_3 and a_5 = areas above the A-G line

Similarly a_2, a_4, and a_6 = areas below the A-G line

Energy in the fly wheel at A=E

At $B = E + a_1$

$C = E + a_1 - a_2$

$D = E + a_1 - a_2 + a_3$

$E = E + a_1 - a_2 + a_3 - a_4$

$F = E + a_1 - a_2 + a_3 - a_4 + a_5$

$G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

Let greatest energy at B and least energy is at E.

Maximum energy = E + a_1

Minimum energy = E + $a_1 - a_2 + a_3 - a_4$

Maximum fluctuation of energy,

?E = maximum energy – minimum energy

= E + a_1 - (E + $a_1 - a_2 + a_3 - a_4$)

CO-EFFICIENT OF STEADYNESS:-

It is the ratio of co-efficient of fluctuation of speed. M =

Co-efficient of fluctuation of energy:-

It is the ratio of maximum fluctuation of energy to the work done per cycle.

Mathematically = C_E = ?

E = work done per cycle

Weight of the fly wheel:-

Let weight of the fly wheel, W= mg

Where m = mass of the fly wheel in kg.

Let K = radius of direction of fly wheel in metre.

I = mass moment of inertia of the fly wheel about its axis of rotation,

i.e; I = m.k^2

And N_2 = maximum and minimum speeds during the cycle in r.p.m

W_1 and W_2 = Maximum and minimum angular speed in radian/second

N =

W =

Cs = [illegible]

Mean kinetic energy of the fly wheel, Em:-

$E_m = \frac{1}{2} I.W^2$

$= \frac{1}{2} m.K^2W^2$

Maximum fluctuation of energy = maximum K.E – minimum K.E

= [illegible]

= [illegible]

= [illegible]

= I.W([illegible]

= $I.\ W^2$? [illegible]

? [illegible] = $I.W^2$ Cs

⇒ I = [illegible]

⇒ mk^2 = [illegible]

⇒ m = [illegible]

I = [illegible]

⇒ I = [illegible]

Problem:

A horizontal cross compound steam engine develops 300 kw at 900 r.p.m. The co-efficient of fluctuation of energy at found turning the diagram is to be 0.1 and the fluctuation of speed is to be kept with +_0.5% of the mean speed. Find the weight of the fly wheel if the radius of gyration is 2 metre.

ANS:-

Given data:

P = 300 KW

N = 90 R.P.M

Ce = 0.1

W = [illegible] = [illegible] = 9.42

Cs = [illegible] 0.1

$W_1 = W_2 = 0.5\% \times$ [illegible]

= 1% of W

= 0.01W

= 0.0942

K = 2 mt

Work done/cycle = [illegible]

= [illegible] = 200000

? [illegible] = E × [illegible]

= 200 × 1000 × 0.1

= 20,000

m = [illegible]

= [illegible] = 5634.66

w = mg

= 5634.66 × 9.81 = 55276.07 = 55.27kg.

Chapter-5

Balancing of Machine

Syllabus:

5.1 Concept of static and dynamic balancing.
5.2 Static balancing of rotating parts.
5.3 Principles of balancing of reciprocating parts.
5.4 Causes and effect of unbalance.
5.5 Difference between static and dynamic balancing

5.1 Concept of static balancing:

A system of rotating masses is said to be in static balance if the combined mass centre of the system lie on the axis of rotation. The given figure shows a rigid rotor revolving with a constant angular velocity wrad/sec.

A no. of masses, say 3, are depicted by point masses at different radii in the same transverses plane. They may represent different kinds of rotating masses such as turbine blades, eccentric disc, etc. if m1, m2 and m3 are the masses revolving at radii r1, r2 and r3 respectively in the sam,e plane then each mass produces a centrifugal force acting radially outwards from axis of rotation.

Dynamic balancing:

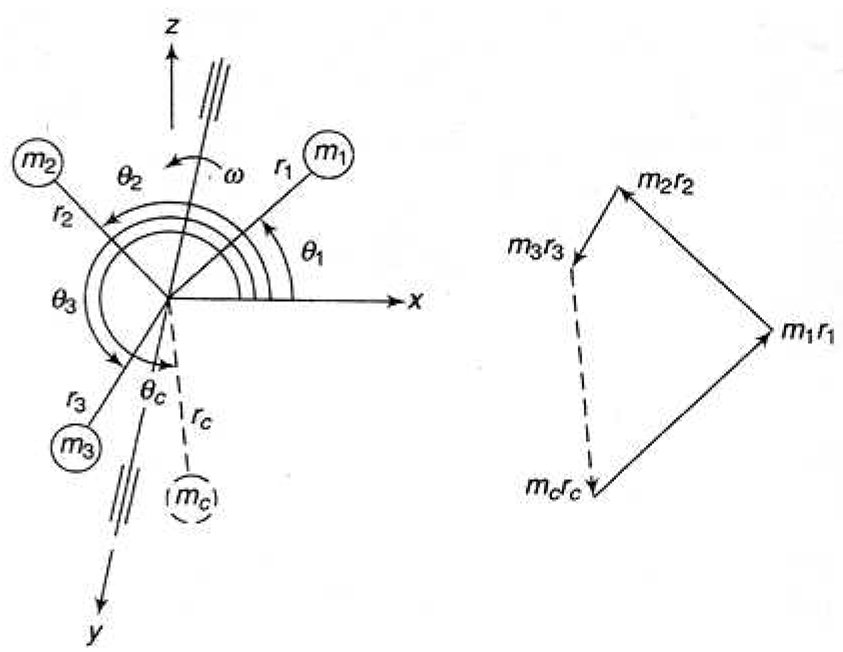
When several masses rotate in different planes, the centrifugal forces, in addition to being out of balance also forms coupkle. A system of rotating masses is in dynamic balance when there doesn't exist any resultant centrifugal force as well as resultant couple:

In the work that follows the product of mr and mri (instead of mrw), usually, have been reform as force and couple respectively as it is more convenient to draws force and couple polygon with these quantities

If m1 m2 are two masses as in figure revolving diametrically opposite to each other in different planes such that m1r1 = m2r2, the centrifugal forces are balanced but an unbalanced couple of magnitude m1r1i = m2r2i is introduced. The couple act in a plane that contains the axis of rotation and the two masses. Thus, the couple is a constant magnitude but variable direction.

5.2 Static balancing of rotating parts:

Consider a number of masses of magnitude m1,m2,m3,m_c at distance of r1,r2,r3,r_e from the axis of the rotating shaft. Let? be the angles of these masses with horizontal line OX.



If σ = 0 (Balancing state)

$M_1r_1w^2+m_2r_2w^2+m_3r_3w^2=0$

? $m_1r_1+m_2r_2+m_3r_3=0$

⇒σ = 0..............................(i)

If not balance state then add m_c .

$m_1r_1w^2 + m_2r_2w^2 + m_3r_3w^2 + m_cr_cw^2 = 0$

⇒ $m_1r_1 + m_2r_2 + m_3r_3 + m_cr_c = 0$

⇒ σ $+m_cr_c=0$...................................(ii)

In components form

σ = 0

σ cos + $m_cr_c\cos{}_c=0$

σ = 0

σ sin + $m_cr_c\sin{}_c=0$

$m_cr_c\cos{}_c=-$ σ cos ....(III)

$m_cr_c\sin{}_c= -$ σ sin

equation (III) (I) &(II) square & add

$(m_cr_c)^2\cos^2{}_c=($σ cos $)^2$

$(m_cr_c)^2\sin^2{}_c=($σ sin $)^2$

⇒ m_cr_c=? ?σ cos ?2 + (σ sin)2 -----------------(4)

Divided equation (II) & (III)

[illegible]

⇒ [illegible]

⇒ [illegible]

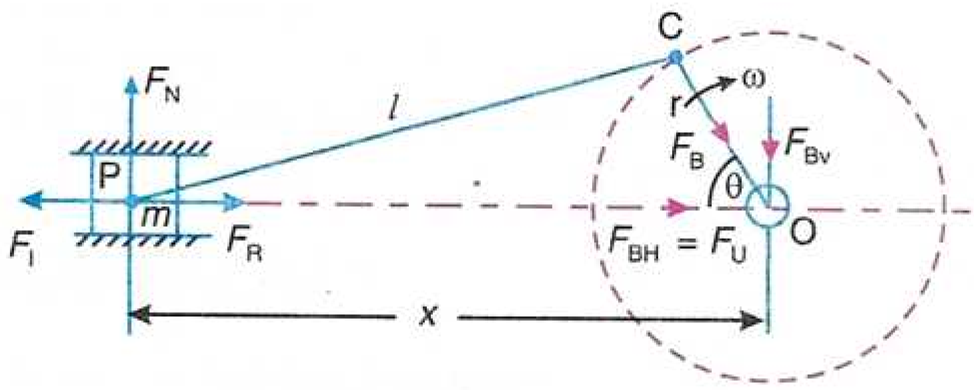
5.3 Principles of balancing of reciprocating parts.

For complete balancing of the reciprocating parts, the following condition must be fulfilled:

1. Primary forces must balance, i.e. primary force polygon is enclosed.
2. Primary couples must balance, i.e. primary couple polygon is enclosed.
3. Secondary forces must balance, i.e. secondary forces polygon is enclosed.
4. Secondary couples must balance, i.e. secondary couple polygon is enclosed.

Usually, it is not possible to satisfy all the above conditions fully for a multicylinder engine. Mostly some unbalanced force or couple would exist in the reciprocating engines.

Balancing of reciprocating masses:



Acceleration of the reciprocating parts is

$a_R =$

where ? = ratio of length & radius l/r.

Inertia force F_I = m (? = F_r)

$F_I = F_R$.

Where F_I

$F_U = F_{BH}$

F_U = m

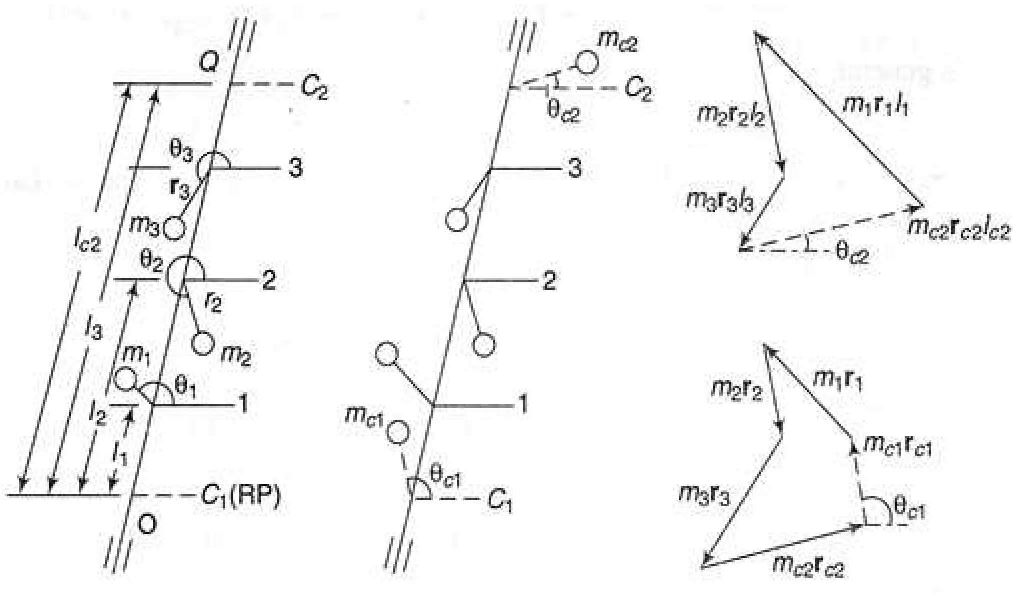
= m

F_P(primary unbalanced forced) F_S(secondary unbalanced force)

F_P(max)= m (? = 90 1)

F_S(max) = m

Balancing of several masses in difference planes:



σ =0

$M_1r_1w^2+m_2r_2w^2+m_3r_3w^2+mc_1rc_1w^2+mc_2rc_2w^2=0$

$\Rightarrow M_1r_1+m_2r_2+m_3r_3+mc_1rc_1+mc_2rc_2=0$

$\Rightarrow$σ $+mc_1rc_1+mc_2rc_2=0$-------------------------------(I)

Taking moments about the point (I)

$M_1r_1l_1w^2+M_2R_2L_2W^2+M_3R_3L_3W^2+Mc_2Rc_2Lc_2W^2=0$

$\Rightarrow M_1r_1l_1+M_2R_2L_2+M_3R_3L_3+Mc_2Rc_2Lc_2=0$

$\Rightarrow$σ $+Mc_2Rc_2Lc_2=0$----------------------(2)

σ

σ

σ

σ

Divided (II) by (I)

⇒tan

Equation (I) in component for

σ = 0

σ + m cos m cos = 0

σ = 0

σ + m sin m sin = 0

cos

m cos - (σ + m cos sin

m sin - (σ + m sin

Both side square & add.

(m cos (σ + m cos

(m sin (σ + m sin

Mc1Rc1=

? ? σ + m cos 2 + (σ + m sin

Divided equation (II) by (I)

tan

Causes & effect of unbalance:

Often an unbalance of force is produce in rotary or reciprocating machinery due to the inertia force associated with the moving masses.

Balancing is the force of designing or modifying machinery that the unbalanced is reducing to an acceptable and possible is eliminated entirely.

In a revolving centre the centrifugal force remains balanced as long as the centre of the mass of the rotor, lies on axis of the shaft when of the mass does lie on the axis or there is an eccentricity, an unbalanced force is produced.

This type of un balanced is very common in case of steam turbines rotors , engine crankshaft, rotary compressor & centrifugal force .

In high speed machinery, the unbalanced force results .there force attracted on the frame by the moving machine members a varying, impact vibration motion on the frame & produced noise . also there are hum an discomfort, determinable effect of machine performance & structural inter grids of the machine foundation .the most common approach lie balancing by disturbing them the mass which may be accomplished by addition or removal of mass from various machine members .

5.5 Difference between static and dynamic balancing:

Static balancing	Dynamic balancing
It would refer to balancing in a single plane	It would refer to balancing in more than one plane
It is also known as primary balancing. It is a balance force due to action of gravity.	It is also known as secondary balancing. It is a balance due to action of inertia forces.
Rotation of fly wheels, grinding wheels, car wheels are treated as static balancing problems as most of their masses concentrated in or very near one plane.	Rotation of shaft of turbo-generator is a case of dynamic balancing problems
Static balance occurs when the centre of gravity of an object is on the axis of rotation	A rotating system of mass is in dynamic balance when the rotation doesn't produce any resultant centripetal force of couple. Here the mass axis is coincidental with the rotational axis.

Chapter-6

Vibration of machine parts

Syllabus:

6.1 Introduction to Vibration and related terms (Amplitude, time period and frequency, cycle)
6.2 Classification of vibration.
6.3 Basic concept of natural, forced & damped vibration
6.4 Torsional and Longitudinal vibration.
6.6 Causes & remedies of vibration.

6.1 Introduction to Vibration and related terms (Amplitude, time period and frequency, cycle)

Amplitude:

It is defined as its maximum displacement of a vibrating body from its equilibrium position.

Time Period:

`It is the time taken by a motion to repeat itself, and is measured in seconds.

Frequency:

Frequency is the number of cycles of motion completed in one second. It is expressed in hertz (Hz) and is equal to one cycle per second.

DEFINITIONS:

I. Free (natural) vibrations elastic vibrations in which there is no friction and external forces after the initial release of the body are known as free or natural vibrations.

II. Damped vibrations when the energy of a vibrating system is gradually dissipated by friction and other resistances, the vibrations are said to be damped. The vibrations gradually cease and the system rests in its equilibrium position.

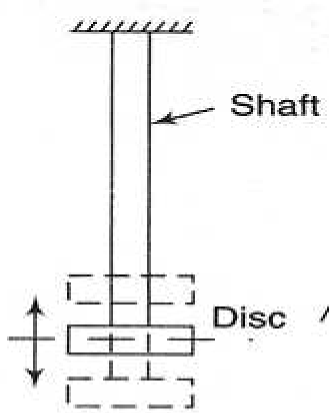
III. Forced vibrations when a repeated force continuously acts on a system, the vibrations are said to be forced. The frequency of the vibrations is that of the applied force and is independent of their own natural frequency of vibrations.

TYPES OF VIBRATIONS

Consider a vibrating body, e.g., a rod, shaft or spring. Figure shows a mass less shaft, one end of which is fixed and the other end carrying a heavy disc. The system can execute the following types of vibrations.

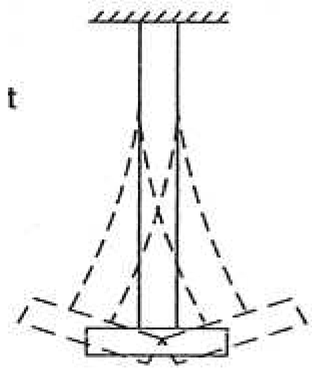
I. **Longitudinal vibrations:**

If the shaft is elongated and shortened so that the same moves up and down resulting in tensile and compressive stresses in the shaft, the vibrations are said to be longitudinal. The different particles of the body move parallel to the axis of the body.



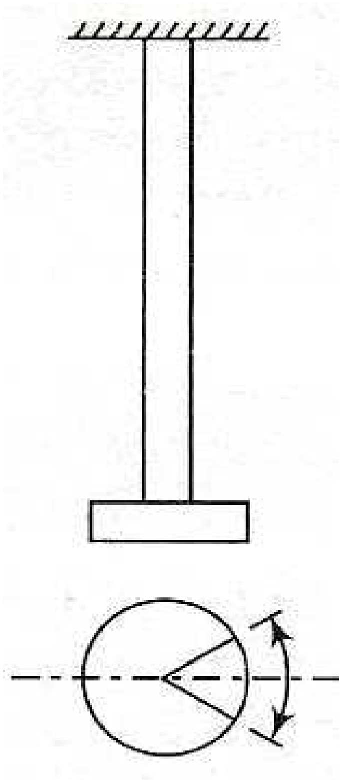
II. **Transverse vibrations**

When the shaft is bent alternately and tensile and compressive stresses due to bending result, the vibrations are said to be transverse. The particles of the body move approximately perpendicular to its axis.



III. **Torsional vibrations:**

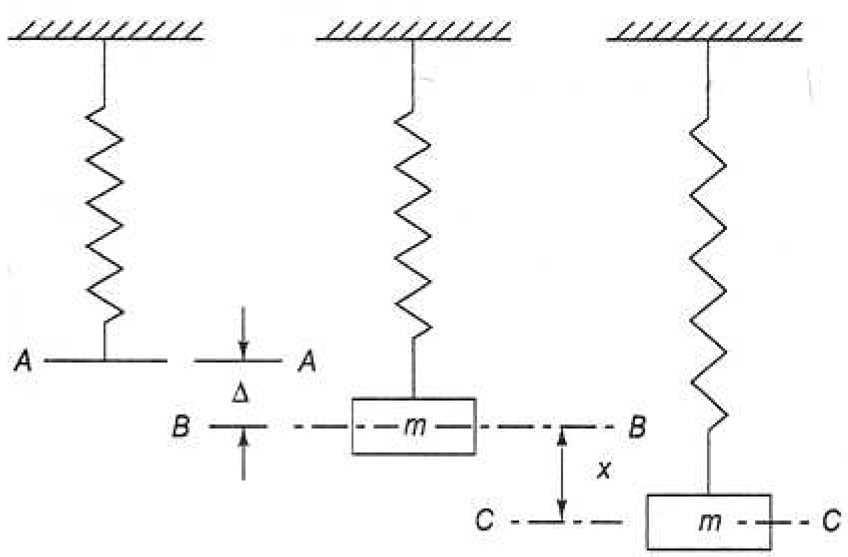
When the shaft is twisted and untwisted and untwisted alternately and torsional shear stresses are induced, the vibrations are known as torsional vibrations. The particles of the body move in a circle about the axis of the shaft.



Free longitudinal vibrations:

The natural frequency of a vibrating system may be found by any of the following method.

Equilibrium method:



It is based on the principle that whenever a vibratory system is in equilibrium, the algebraic sum of forces and moments acting on it is zero. This is in accordance with D'almbert's principle that the sum of the inertia forces and the external forces on a body in equilibrium must be zero.

Figure shows a helical spring suspended vertically from a rigid support with its free end at A-A.

If a mass m is suspended from the free end, the spring is stretched by a distance Δ and B-B becomes the equilibrium position thus Δis the static deflection of the spring under the weight of the mass m.

Let s = stiffness of the spring under the weight of the mass m.

In the static equilibrium position,

Upward force = downward force

S× ? = mg

Now, if the mass m is pulled father down through a distance x, the forces acting on the mass will be

Inertia force = m (upwards)

Spring force(restoring force) = sx (upwards)

(x is downward and thus velocity downwards)

As the sum of the inertia force and the external force on the body in any direction is to be zero (D'alembert's principle),

m 0

If the mass is released, it will start oscillating above and below the equilibrium position. The oscillation will continue for ever if there is no frictional resistance to the motion.

The above equation can be written as

?

This is the equation of a simple harmonic motion and is analogous to

The solution of which is given by

x = A sin

Where A and B are the constants of integration and their values depend upon the manner in which the vibration starts. By making proper substitutions, other forms of the solution can also be obtained as follos:

- By assuming A = X cos and B = X sin

x = x(sin t.cos + cos t.sin)

= xsin( )

Where X and are the constants and have to be found from initial conditions.

- By assuming A= X sin and B = X cos

$$x = X(\sin\ t.\sin + \cos\ t.\cos\ )$$

$$= x = X.\cos(\ )$$

Where X and are the constants and have to be found from initial conditions. The above solutions show that the system vibrates with frequency.

$$= ?\ \frac{}{}.$$

Which is known as the natural circular frequency of vibration. As one cycle of motion is completed in an angle 2, the period of vibration is

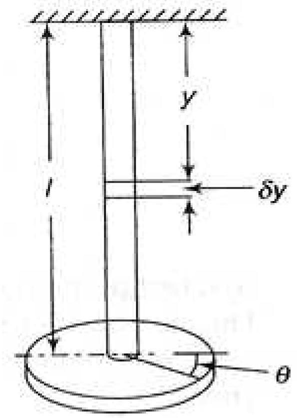
$$T = \frac{}{} = 2 .?\ \frac{}{}$$ and, natural linear frequency of the vibrating system,

$$f_n = \frac{}{} = \frac{}{}\ \frac{}{}.$$

In general, the words circular or linear are not used in natural circular frequency or in natural linear frequency. Both are known as natural frequencies of vibration and are distinguished by their units.

Free torsional vibrations (single rotor)

Consider a uniform shaft of length/rigidly fixed at its upper end and carrying a disc of moment of inertia/ at its lower end. The shaft is assumed to be massless. If the disc is given a twist about its vertical axis and then released, it will start oscillating about the axis and will perform torsional vibrations.



Let:

θ = angular displacement of the disc from its equilibrium position at any instant

q = torsional stiffness of the shaft

= torque required to twist the shaft per radian within elastic limits

= $\frac{GJ}{l}$

where G = modulus of rigidly of the shaft material

J = poler moment of inertia of the shaft cross-section

At any instant, the torques acting on the disc are

- Inertia torque = $-I\ddot{\theta}$
- Restoring torque(spring torque) = $-q\theta$

Negative signs have been used as both of this torques act opposite to the angular displacement. For equilibrium, the sum of all torques acting on the disc must be zero.

Therefore, [illegible] = 0

Or [illegible] = 0

This is the equation of simple harmonic motion.

[illegible] = [illegible]

[illegible] = [illegible]

Causes & remedies of vibration:

Causes of vibration:

Some of the important causes of vibration in machines are listed below

1. Unbalanced reciprocating machine parts
2. Unbalanced rotating machine parts
3. Incorrect alignment of the transmission elements such as coupling etc
4. Use of simple spur gears for power transmission
5. Warm-out teeth of the gears of the power transmission
6. IOmpact taking parttsd of ther machine of axplosion or impact of working fluids of prime movers
7. Loose transimission bekts and chains.
8. Loose fastenings of the moving parts.
9. Vibration waves from other sources and machines installed neardy, due to improper isolatrion of vibrations from them.
10. Due to more material constact such as bases plates on the foundations for the pedestal bearings.
11. Non-rigid machine foundations due to lack of compact soil below, causing settlement of machine compounents.

Remedies of vibration:

Although it is impossible to eliminate the vibrations, yet these can be reduced by adopting various remedies, some of the remedies are listed below:

1. Partial balancing of reciprocating masses.
2. Balancing of unbalanced rotating masses.
3. Using helical gears instead of spur gases.
4. Proper tightening and locking of fastening and periodically ensuring it again.
5. Correcting the mis-0alingnmemnt of rotating components and checking it from time to time.
6. Timly replacement of work-out moving parts, slides and bearings with excessive clearance.
7. Isolating vibrations from other machines and sources by providing vibration insulation pads in the machine foundations.
8. Making machine foundatioins on compact ground and making them sufficiently strong, so that they do not yield or settle under the load of the machine.