CHAPTER-1

1.1 TRANSDUCERS

Transducer is a device which transforms energy from one form to another. It also converts any non-electrical quantity to electrical quantity.

i.e. (1) Mechanical energy to Electrical signal
(2) Heat to Electrical signal.
(3) Liquid level to Electrical Signal.
(4) Flow rate to Electrical Signal.

There are two types of transducers:

(1) Electrical Transducers
(2) Mechanical Transducers

Electrical Transducers

Non-electrical quantities such as temperature, pressure, displacement, speed, fluid flow cannot be measured directly. The electrical Quantities such as voltage, current, resistance, inductance capacitance can be measured directly. For measurement of non-electrical quantities these quantities are to be converted into the electrical quantities and then measured. This function of converting non-electrical quantities to electrical quantities is accomplished by a device called as the Electrical Transducers.

Advantages of Electrical Transducers

- The electrical output from the transducer can be amplified into any desired level of voltage and current.
- Very small power is required for the controlling of the electrical and electronic systems.
- Frictional effect is reduced to minimum.
- The output can be measured at a distance from the sensing medium through wired of wireless medium.

Disadvantages of Electrical Transducers

- The electrical transducers are comparatively costly than the other types of transducers.
- They have low reliability in comparison to that of the mechanical transducers.

1.2 CLASSIFICATION OF TRANSDUCERS

Classification of Transducers on the basis of method of application

On the basis of method of application, the transducers can be classified as:

(1) Primary Transducers
(2) Secondary Transducers

Primary Transducers

When the input is sensed by transducer and the physical phenomenon is converted into electrical signal then such transducers is called as primary transducers.

e.g.: Thermistors: It senses the temperature directly and causes change in resistance with the change in temperature.

Secondary Transducers

When the input is first sensed by a sensor, output being of some form other than input signal is given to the transducer for conversion into electrical form. Such transducer is called as secondary transducers. It consists of two parts: sensing element and transduction element.

E.g.: LVDT (Linear Variable Differential Transducers): Displacement is converted to the electrical signal by LVDT.

Sensing element: It corresponds to the change in physical phenomenon.

Transduction element: It changes the output of the sensing element into electrical signal.

Classification of Transducers on the basis of energy conversion:

On the basis of energy conversion, the transducers can be classified as:

(1) Active Transducers
(2) Passive Transducers

Active Transducers

An active transducer does not require any external power source for its operation. They develop their own voltage or current. An active transducer can generate an electrical signal directly in response to the physical quantity to be measured.

E.g.: Thermocouple, Tacho-generator, Photovoltaic cell.

Passive Transducers

These transducers require an external power source for its conversion. In these tranducers electrical parameters such as resistance, Inductance, Capacitance changes and it causes a change in voltage and current.

E.g.: Resistive transducer, Inductive transducer and capacitive transducer.

Classification of Transducers on the basis of nature of output signal

On the basis of energy conversion, the transducers can be classified as:

(1) Analog Transducers
(2) Digital Transducers

Analog Transducers

It converts input signal to output signal which is continuous function of time.

E.g.: Strain gauge, thermocouple, strain gauge.

Digital Transducers

It converts input signal into output signal in the form of pulses which gives discrete output. These are more popular in use in the form of digital measuring instruments.

1.3 Resistive Transducers

In resistive transducers, resistance changes due to change in some of the physical phenomenon.

Resistance of any metal conductor, $R = \rho \frac{L}{A}$

Where ρ = Specific resistance of the material (resistivity)

L = Length of the wire

A = Cross-sectional Area of the wire

If there is a change in ρ, L, A then there is a change in resistance. Thus it can be said that input signal to the transducer causes the variation in the resistances by changing the value of any one quantity of ρ, L, A.

E.g.: Potentiometer: Length varies so there is a change in resistance.

Strain Gauges: When the gauge is strained then there is a change in cross sectional area and hence there is a change in resistance.

For temperature measurement there is a change in resistivity 'ρ'.

1.3.1. Potentiometer

Potentiometer is a very simple device where there si a change in output voltage due to change in displacement. It is a passive transducer which requires an external power source for its operation. It is a very cheap form of transducer and is very widely used.

It converts linear displacement to the voltage by change in resistance and also rotational displacement to voltage by change ion resistance.

It is of the following types:

- Linear potentiometer.
- Rotational Potentiometer
- Helipot

Linear Potentiometer

Linear potentiometers are used for the measurement of displacement. It is simple and cheap. Its output voltage is a function of linear displacement.

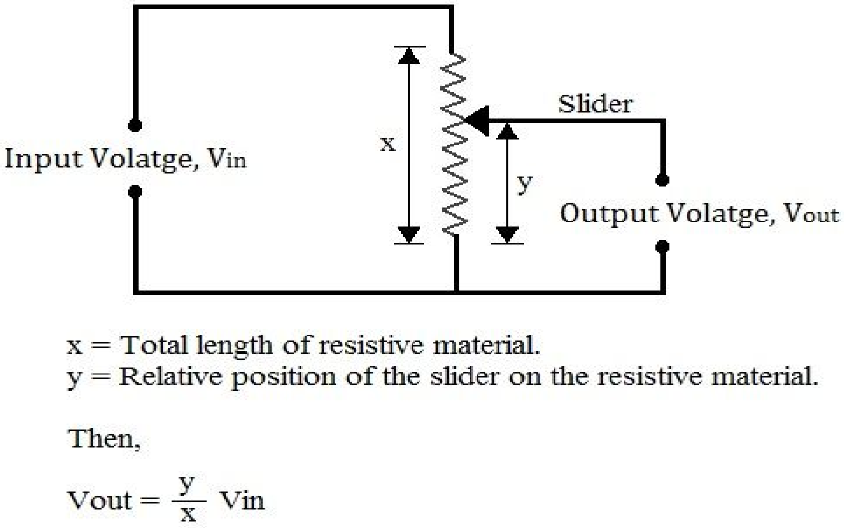


Fig. 1.1: Potetiometric arrangemet

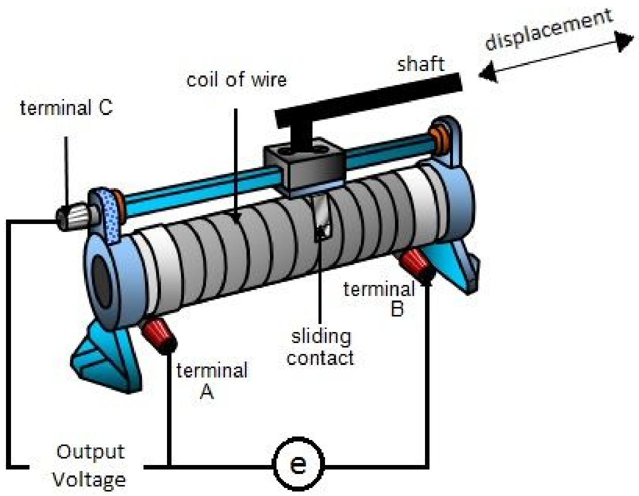


Fig. 1.2: Potentiometer connection using a rheostat

Construction:

It consists of a resistance element provided with a sliding contact called as wiper. The resistance element is a wire made up of platinum or nickel alloy. This resistance element is wound uniformly throughout the length of the sliding contact. Displacement of the slider/wiper determines the change in the voltage drop related to the position of slider. It is mostly used in the laboratory demonstrations and is not suitable for industrial use.

Rotary Potentiometer:

This kind of potentiometer operates in the same way as that of the linear potentiometer. In rotary potentiometer the resistance element is connected a circular manner in case of rotary potentiometer. This kind of potentiometer has a rotary movement and is useful in the measurement of angular displacement. The angular measurement can be measured between 1^0 and 357^0.

When the motion of the wiper is in both translational and rotational way then the potentiometer is called as helipot.

Advantage of potentiometer:

- It can measure large displacement.
- It has high electrical efficiency.
- It is simple in construction.
- It is cheap and easy to operate.

Disadvantages of potentiometer:

- Sliding contact may wear out and get contaminated or generate noise.
- They require large force to move their sliding contact.

1.3.2. STRAIN GAUGES

If a metal conductor is stretched or compressed, its resistances changes due to the change in both length and diameter of the conductor.

The change in the value of the resistance by straining of the gauge may be partially explained. As the gauge is under a positive strain, the length of it increases and its area of cross section decreases. Since the resistance of the conductor is proportional to the length and inversely proportional to its area of cross section, the resistance of the gauge increases with positive strain.

Let us consider a strain gauge made up of circular wire

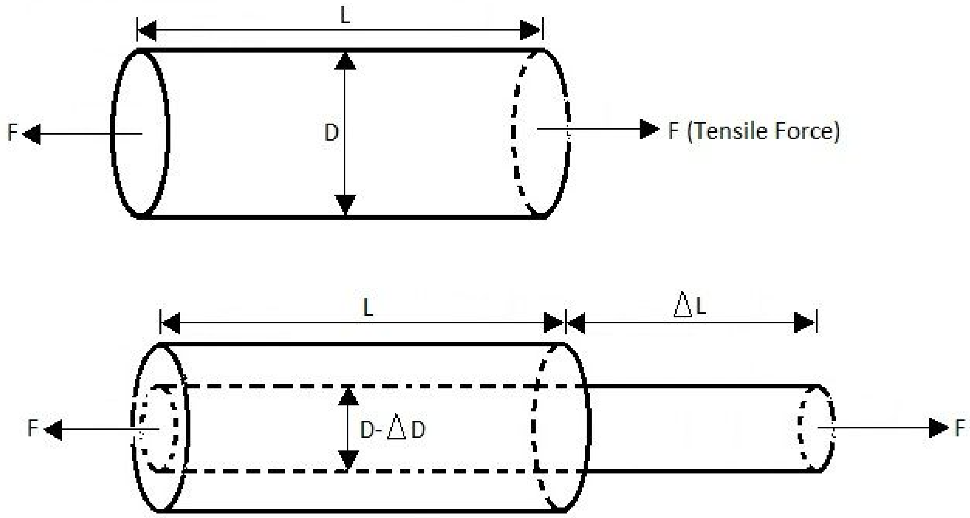


Fig. 1.3: Change in dimensions of a strain gauge due to force 'F'

L = Original length of wire

D = Original Diameter of wire

F = Tensile Force

ΔL = Change in length
ΔD = Change in Diameter
ΔR = Change in Resistance
ΔA = Change in cross sectional area

We know,
$$R = \rho \frac{L}{A}$$
Differentiating with respect to stress s,
$$\frac{dR}{ds} = \frac{\rho}{A}\frac{\partial L}{\partial s} - \frac{\rho L}{A^2}\frac{\partial A}{\partial s} + \frac{L}{A}\frac{\partial \rho}{\partial s}$$
Dividing by $R = \rho \frac{L}{A}$
$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{1}{A}\frac{\partial A}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s}$$
Per unit change in length = $\frac{\Delta L}{L}$

Per unit change in area = $\frac{\Delta A}{A}$

Per unit change in Resistivity = $\frac{\Delta \rho}{\rho}$

$$A = \frac{\pi}{4}D^2$$

$$\frac{\partial A}{\partial s} = 2\frac{\pi}{4}D\frac{\partial D}{\partial s}$$

$$\frac{1}{A}\frac{\partial A}{\partial s} = \frac{\left(\frac{2\pi}{4}\right)}{\left(\frac{\pi}{4}\right)}\frac{D}{D^2}\frac{\partial D}{\partial s} = \frac{2}{D}\frac{\partial D}{\partial s}$$

We know,

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{1}{A}\frac{\partial A}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s}$$

Poisson's ratio, $\nu = \frac{Lateral\ Strain}{Longitudinal\ Strain} = -\frac{\frac{\partial D}{d}}{\frac{\partial L}{L}}$

$$\frac{\partial D}{D} = -\nu \times \frac{\partial L}{L}$$

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \nu\frac{2}{L}\frac{\partial L}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s}$$

The above relation can be written as:

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\nu\frac{\Delta L}{L} + \frac{\Delta\rho}{\rho}$$

Gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

$$G_f = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}}$$

Therefore, $G_f = 1 + 2\nu + \frac{\Delta\rho/\rho}{\Delta L/L}$

$$\frac{\Delta L}{L} = \epsilon = \text{Strain}$$

Thus $G_f = 1 + 2\nu + \frac{\Delta\rho/\rho}{\epsilon}$

Unbonded Metal Strain Gauges

This gauge consists of a wire stretched between two points in a insulating medium such as air. The wires may be made up of copper-nickel, chrome-nickel or nickel-iron alloys. The wires are tensioned to avoid buckling when they experience a compressive force.

The unbounded metal wire gauges used almost exclusively in transducer applications, employ preload resistance wires connected in a wheat-stone bridge. At initial period, the strains and resistances of the four arms are normally equal, with this result, output voltage of the bridge, $e_0 = 0$. When displacement occurs, it increases tension in two wires and decreases in the other two, thereby increase the resistance of the two wires which are in tension and decreasing the resistance of the two wires. This causes an unbalance of the bridge producing an output voltage which is proportional to the input displacement and hence to the applied pressure.

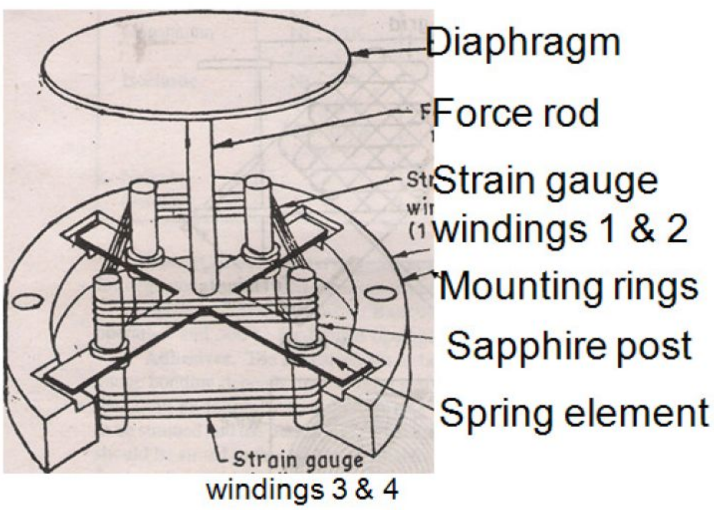


Fig. 1.4: Constructional diagram of Unbonded metal strain gauge

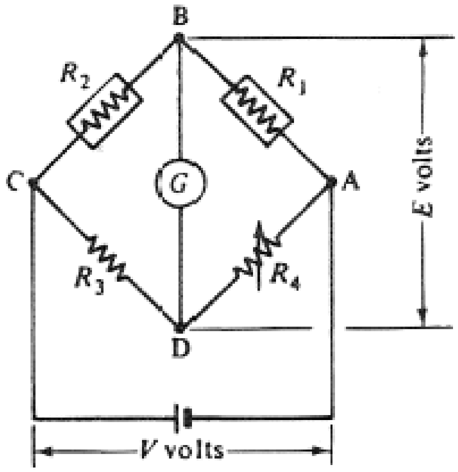


Fig. 1.5: Schematic diagram of unbonded metal strain gauge

Bonded Wire Strain Gauges

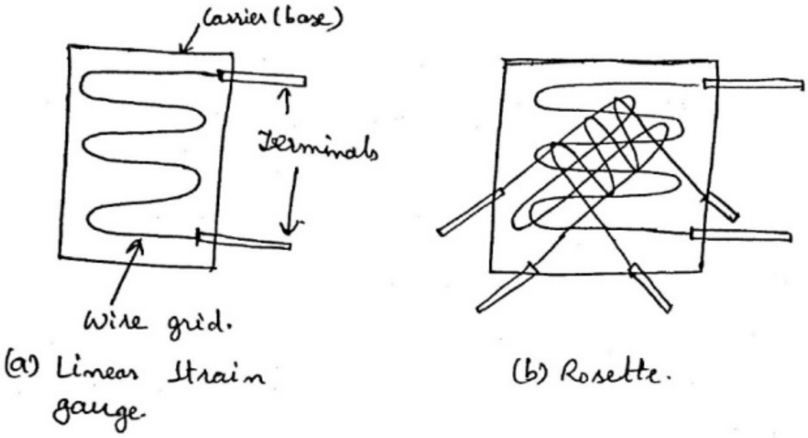
It is used for both stress analysis and for constructing transducers.

A resistance wire strain gauge consists of grid of fine resistances wire of about 0.025mm in diameter or less. The grid is cemented to carrier (base) which may be thin sheet of paper, a thin sheet of Bakelite or sheet of Teflon. The wire is covered on top with a thin sheet of material so as to prevent it from any mechanical damage. The spreading of the wire permits an uniform distribution of stress over the grid. The carrier is bonded with an adhesive material to the specimen under study. The wires cannot buckle as they are embedded in a matrix of cement and hence faithfully follow both the tensile and the compressive strains of the specimen. The materials and the wire sizes used for the banded wire strain gauges are the same as used for the unbonded wire strain gauges.

It is desirable that resistance wire strain gauges should have the following characteristics:

(1) Strain gauge should have a high value of gauge factor G_f. A high gauge factor indicates high sensitivity.

(2) The resistances of the strain gauges should be as high as possible since this minimizes the effects of undesirable variations of resistance in the measurement circuit.



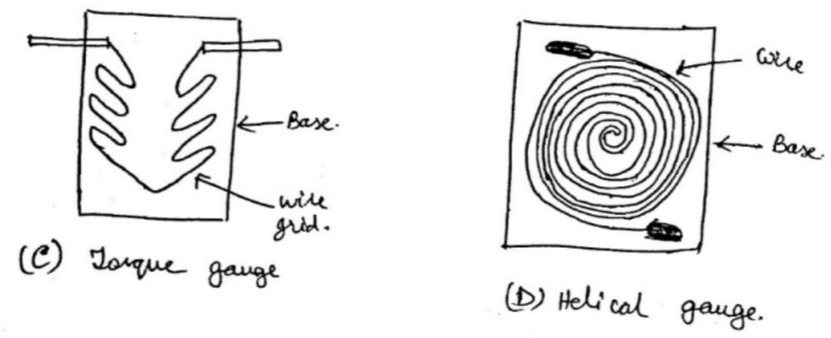


Fig. 1.6: Different types of bonded strain gauge

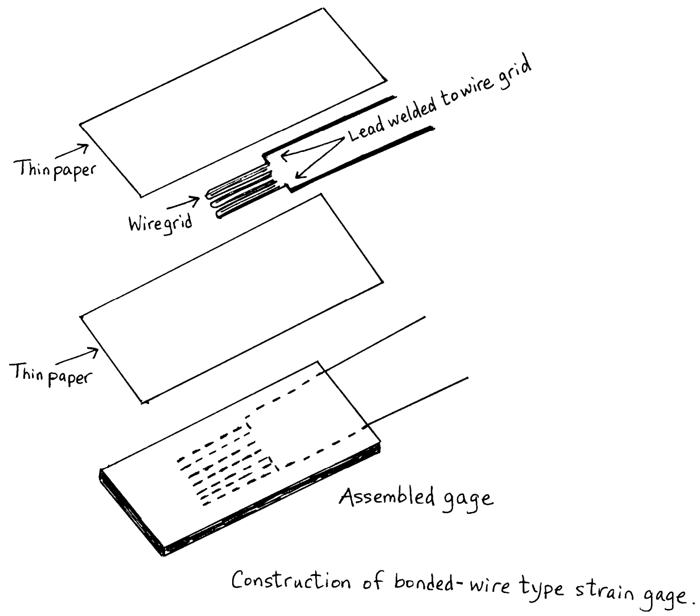


Fig. 1.7: Construction of Bonded strain gauge

1.3.3. PLATINUM RESISTANCE THERMOMETER

The resistance type thermometer bulbs are sensing elements in the form of the wires or foils. The films deposited on insulating surfaces are also used for temperature sensing. In the wire type, the arrangement are commonly a helical coil wound as a double wire to avoid inductive effects. The laboratory type resistance thermometers have the temperature sensing element wound on a cross mica former and enclosed in Pyrex tube shown in fig. 1.

The industrial type of thermometer shown in fig. 2. Here the former is of grooved ceramic and the wire is being protected by a glass coating or by a stainless steel tube. The element is normally sealed in glass when used for temperatures upto 150^{o}C and ceramic for use in temperatures upto 850^{0}C.

Resistive elements are also made up of thin etched grids of metal foils similar in shape of the foil type strain gauges. They are constructed of the platinum and may be bonded to plastic booking for attachment to a surface.

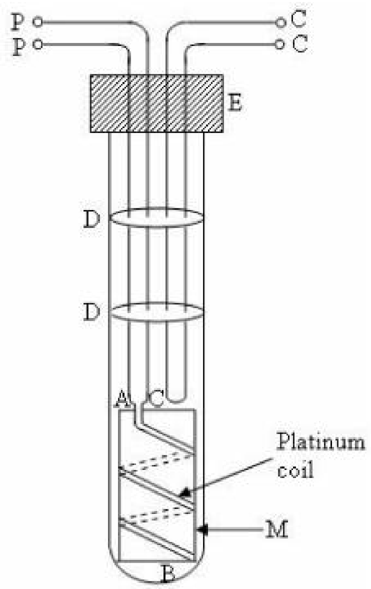


Fig. 1.8: Schematic diagram of Platinum Resistance Thermometer

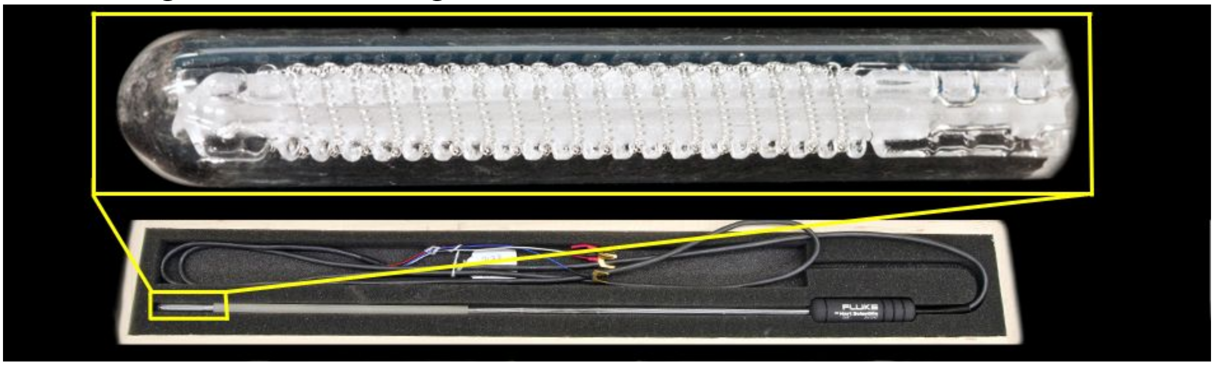


Fig. 1.9: Platinum Resistance thermometer

1.3.4. THERMISTOR

Thermistors are general composed of semi-conductors. Most thermistors have a negative temperature coefficient i.e. their resistance decreases with increase of temperature. This allows thermistors circuits to detect very small changes in temperature which could be observed using a RTD or a thermocouple.

Thermistors are widely used to measure temperature ranging from -600C to 150C. Resistance of thermistors ranges from 0.5Ω to 0.75MΩ.

Construction of thermistors:

These are composed of sintered mixture of metallic oxides such as manganese, nickel, cobalt, copper, iron and uranium. They are available in variety of shapes and sizes. They may be in the form of beads, rods and disks. Thermistors in the form of beads are the smallest in size.

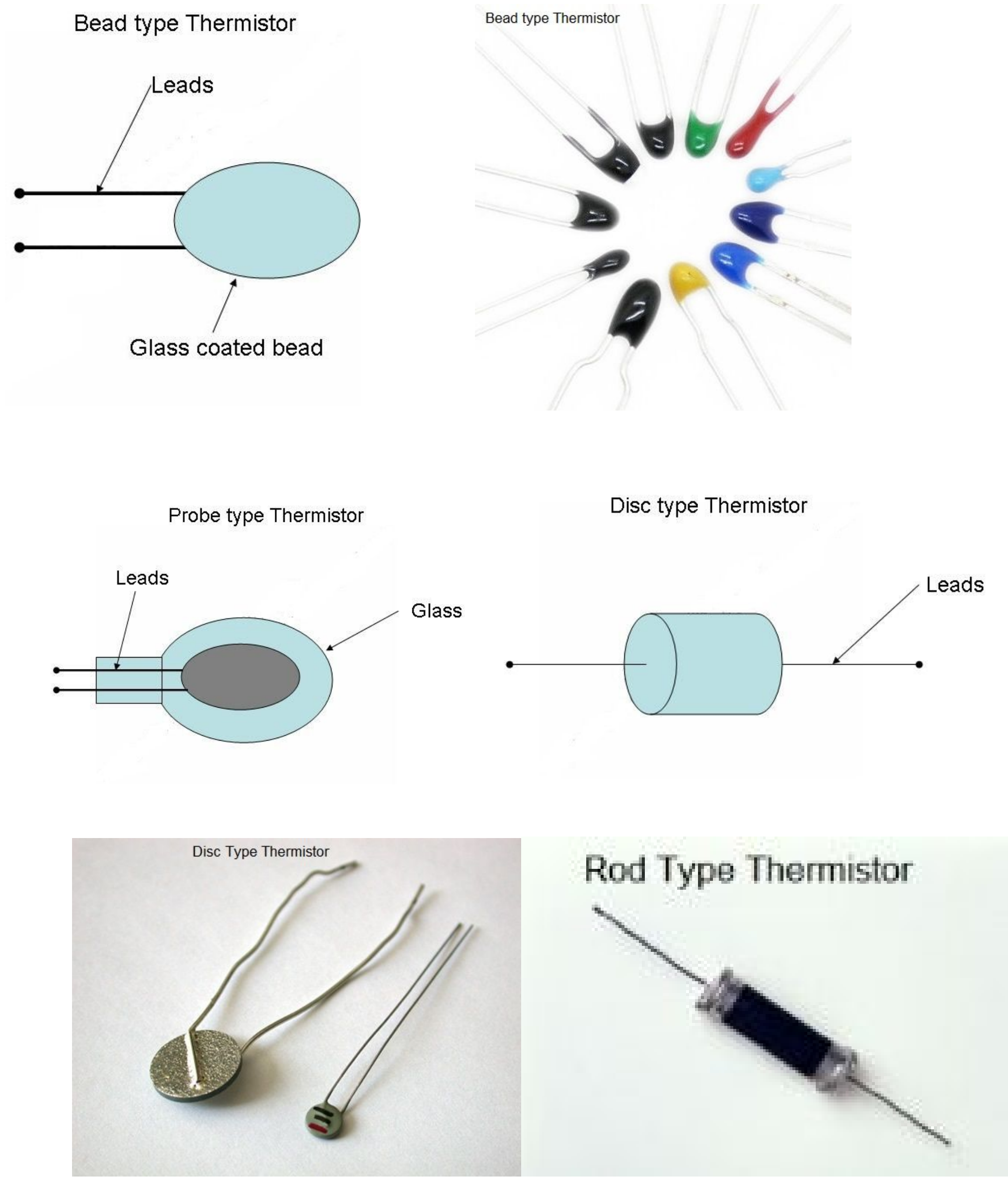


Fig. 1.10: Different types and construction of Thermistors

Resistance temperature characteristics of thermistor

It is given by,

$$R_{T1} = R_{T2} exp[\beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right)] .$$

R_{T1} =Resistance of the thermistor at temperature T_1 (0K)

R_{T1} =Resistance of the thermistor at temperature T_1 (0K)

β = Constant, depending upon the material of thermistor.

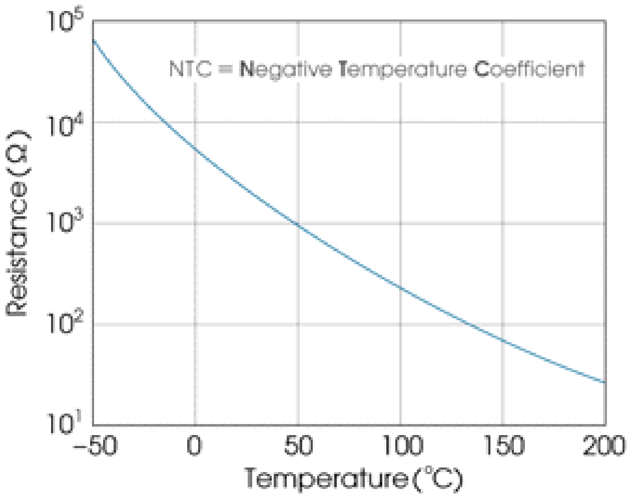


Fig. 1.11: Resistance temperature characteristics of thermistor

Thus a thermistor has a very high negative temperature coefficient of resistance making it an ideal temperature transducer. Between -100^{0}C to 400^{0}C, the thermistor changes between its resistivity from 10^4 to 10^{-4}Ωm which explains high sensitivity of thermistors for measurement of temperature.

1.4. INDUCTIVE TRANSDUCER

The most commonly used inductive transducer is Linear Variable Differential Transducer (LVDT).

LVDT is widely used for translating linear motion and displacement into electrical signals. It is a passive transducer.

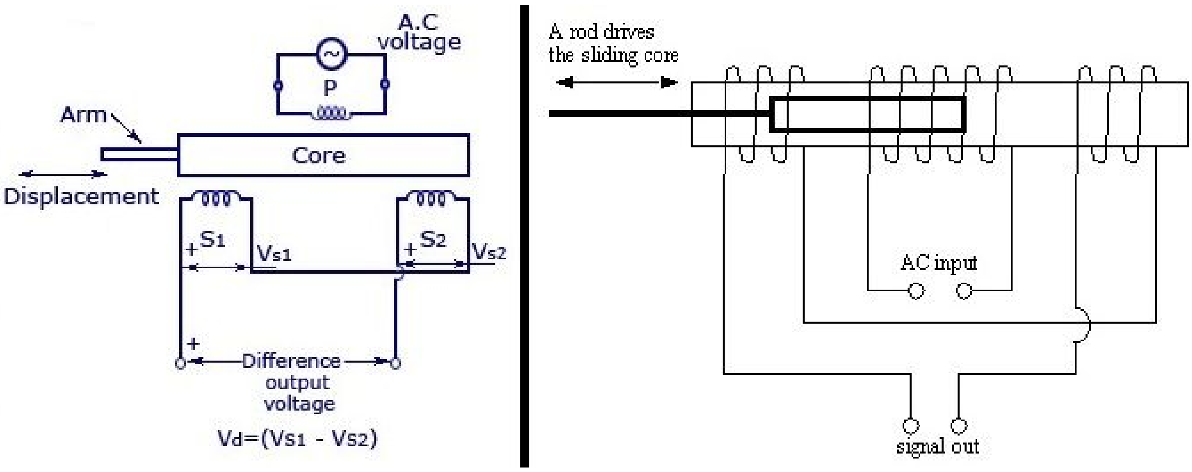


Fig. 1.12: Schematic diagram of LVDT

Construction:

LVDT consists of two windings: (a) One Primary Winding, P (b) Two secondary winding S_1& S_2. These two windings are wound on hollow cylindrical former known as 'bobbin' which is made up of either non-magnetic material or insulating material.

The secondary windings S1 & S2 have equal number of turns and are placed on either side of the primary winding. Primary winding is connected to the AC source.

A moveable cylindrical shape soft iron core is attached to the sensing element of the transducer. The core slides freely within the hollow portion of the bobbin. Core is made up of nickel iron alloy to reduce the eddy current loss. The displacement to be measured is attached to the soft iron core. In order to get a differential output voltage, the two single voltage signals V_{s1} and V_{s2} from the two secondary windings S_1 and S_2 are connected in series in a particular fashion. Such a circuit connection has been adopted to ensure that we get a difference of the two output voltages V_{s1} and V_{s2} and not a summation.

Working:

Any physical displacement of the core causes the voltage of one secondary winding to increase while simultaneously reducing the voltage in the other secondary winding. The difference of the two voltages appear across the output terminal of the transducer and gives a measure of the physical position of the core and hence the displacement.

When the core is at the normal position the flux linking with both secondary windings is equal and hence equal EMFs are induced. So the net output is $V_0 = V_{s1} - V_{s2} = 0$.

If the core is moved to the left of the null (normal) position, then more flux gets linked with the winding 'S_1' and less with the winding 'S_2'. So $V_{s1} > V_{s2}$. Hence we get a positive voltage at the output.

Similarly, if the core is moved to the right of the null (normal) position, then more flux gets linked with the winding 'S_2' and less with the winding 'S_1'. So $V_{s1} < V_{s2}$. Hence we get a negative voltage at the output.

So when

$V_{s1} > V_{s2}$ *then* $V_0 = V_{s1} - V_{s2} = +ve$

$V_{s1} < V_{s2}$ *then* $V_0 = V_{s1} - V_{s2} = -ve$

The difference of the output voltage of the secondary windings gives the amount of displacement.

Applications:

It is widely used for the measurement of displacement ranging from few mm to few cm.

Advantages:

- Its output is very high. So no need of amplifying the output.
- The device consumes less power.
- It is very simple, light in weight and easy to maintain.
- It can tolerate high degree of shock and vibration.

Disadvantages:

- These devices are very sensitive to stray magnetic field.

1.5. CAPACITIVE TRANSDUCER

It is a device in which the capacitance is varied by the non-electrical quantity being measured. If there is a change in a physical quantity such as force displacement and pressure then there is a change in capacitance.

Operating principle of capacitive transducer is based on the principle of parallel plate capacitor.

Capacitance, $C = \frac{\epsilon_0 \epsilon_r A}{D}$

Where C = Capacitance (in F)

A = Overlapping area of the plate (in m^2)

D = Distance between the plates

ϵ_r =Relative permittivity

ϵ_0 = Permittivity of free space

Capacitive transducer work on the principle of the change of capacitance which may be caused by:

(1) Change in overlapping area (A).
(2) Change in distance (D) between the two plates.
(3) Change in dielectric constant (ϵ).

All these changes are caused due to changes in physical quantities like pressure, force which causes change in capacitance and hence the non-electric quantities get changed to electrical quantities.

Transducer using change in area of plate

For cylindrical Tube:

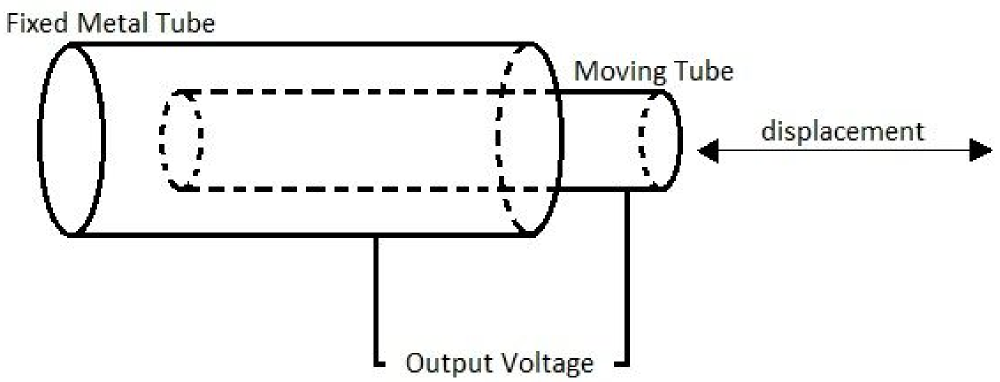


Fig. 1.13: Change in Capacitance due to change in area of the cylindrical plates

In this diagram the moving tube is inside a fixed tube. The movement in the moving tube causes a change in the overlapping area (A) due to which the capacitance changes and hence there is a change in the output voltage.

$$C = \frac{\epsilon_0 \epsilon_r A}{D}$$

$$C \; \alpha \; A \text{ and } C \; \alpha \frac{1}{D}$$

For parallel plate:

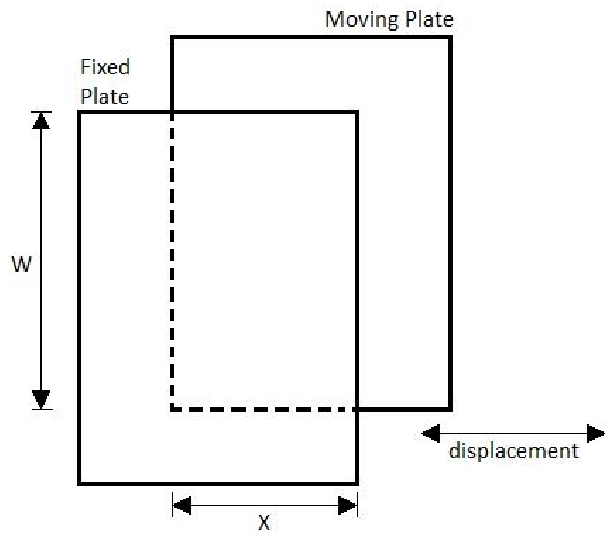


Fig. 1.14: Change in Capacitance due to change in area of the parallel plates

In this diagram the capacitance changes due to the changes in the overlapping area(A). The overlapping area changes due to the displacement in the moving plate. Currently the overlapping area (A) is $W \times x$. Thus the capacitance is given by:

$$C = \frac{\epsilon_0 \epsilon_r (W \times x)}{D}$$

Hence when there is a change in W or x then there is a change in the capacitance due to the change in the overlapping area (A).

Transducer using change in distance between the two plates

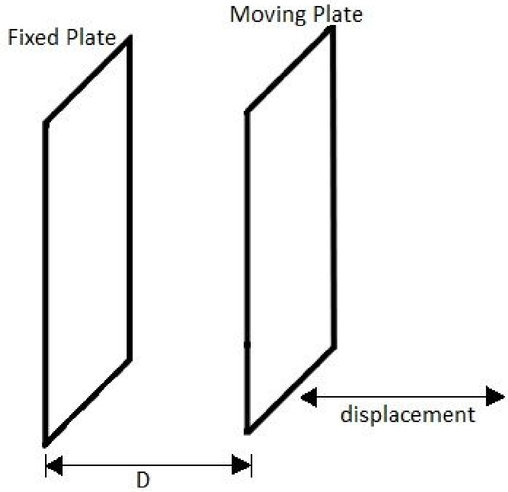


Fig. 1.15: Change in Capacitance due to change in distance between the plates

In this mechanism if there is a change in the distance between the two plates then there is a change in capacitance.

Since we know,

$$C = \frac{\epsilon_0 \epsilon_r A}{D}$$

Hence, from the formula it can be observed that,

$$C \, \alpha \, \frac{1}{D}$$

So it is quite clear that the capacitance is inversely proportional to the distance between the plates. So when the distance increases then there is a decrease in capacitance and vice versa.

Application:

- Capacitive transducers are used for measurement of small displacement.
- These are used for linear displacement.

Advantages of capacitive transducer:

- Very small force is required to operate the system.
- It is highly sensitive.

Disadvantages of capacitive transducer:

- Capacitive transducers are very sensitive to vibrations and temperature.
- Metallic parts of capacitor must be insulated from each other.
- Performance of capacitive transducers is affected by humidity, dirt and other contaminations.

1.6. PIEZOELECTRIC TRANSDUCER

A piezoelectric material is one in which an electric potential appears across certain surfaces of a crystal if the dimensions of the crystal are changed by the application of a mechanical force. This potential is produced by the displacement of charges. The effect is reversible i.e. conversely, if varying potential is applied to the proper axis of the crystal, it will change the dimensions of the crystal thereby deforming it. The effect is known as piezo-electric effect.

Common piezo-electric materials are: Rochelle salt, ammonium dihydrogen phosphate, Lithium sulphate, dipotassiumtartarate, quartz and ceramics A and B. except for quartz and ceramics A and B, the rest are man-made crystals grown from aqueous solution under carefully controlled conditions.

The materials that exhibit a significant and useful piezo-electric effect are divided into two categories:

(1) Natural group – quartz and Rochelle salt.

(2) Synthetic group – Lithium sulphate, Ethylene diaminetartarate

The piezo-electrc effect can be made to respond to the mechanical deformation s of the material. It is used for converting mechanical motion to electrical signals, which can be thought like charge generator and a capacitor.

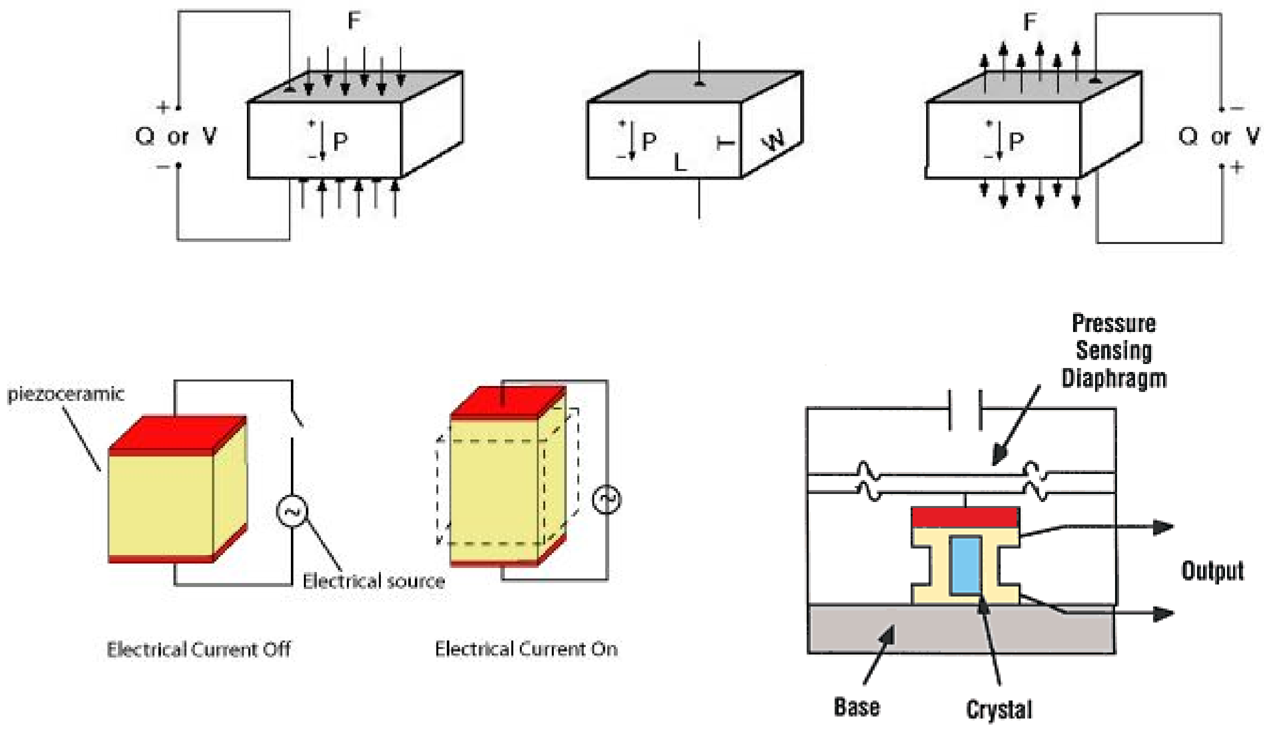


Fig 1.16: Schematic of Piezo-electric Transducers

Fig. 1.17: Piezoelectric Transducer

Mechanical deformation generates a charge and this charge appears as voltage across the electrodes voltage, $E = \frac{Q}{C}$

A tensile force produces a voltage of one polarity while a compressive force produces a voltage of opposite polarity.

Charge, $Q = d \times F \quad ...(1)$

d = Charge sensitivity of the crystal (Constant for a given crystal) [Unit: C/N]

F = Applied force(N)

$$F = \frac{AE}{t}\Delta t \;\; (N) \qquad \ldots (2)$$

A = Area of crystal (m^2)
t = Thickness of crystal (m)
E = Young's modulus(N/m^2)

$$E = \frac{Stress}{Strain} = \frac{F}{A}\frac{1}{\Delta t/t} = \frac{Ft}{A\Delta t} \;\; (N/m^2)$$

Also, $A = w.l$ Where w = Width of the crystal
l = Length of the crystal

From equations (1) and (2),

$$Q = dAE(\Delta t/t)$$

Output voltage, $E_0 = Q/C_p$
C_p = Capacitance between electrodes
Also, $C_p = \epsilon_r \epsilon_0 A/t$

$$E_0 = \frac{Q}{C_p} = \frac{dF}{\epsilon_r \epsilon_0 A/t} = \frac{dt}{\epsilon_r \epsilon_0}\frac{F}{A}$$

$F/A = P$ = Pressure or stress inN/m^2.

$$E_0 = \frac{dt}{\epsilon_r \epsilon_0}.P$$

$E_0 = g.t.P$

Where $E_0 = \frac{d}{\epsilon_r \epsilon_0}$ ... (3)

g = Voltage sensitivity of the crystal (This is a constant for a given crystal. Its unit isVm/N)

$$g = \frac{E_0}{t.P} = \frac{E_0/t}{P} = \frac{\epsilon}{P}$$

Where $\epsilon = \frac{E_0}{t}$ = electric field

$$g = \frac{eectric\ field}{stress} = \frac{\epsilon}{P}$$

Crystal voltage sensitivity, g, can be defined as the ratio of electricfield intensity to pressure (or stress). The units of g areVm/N.

From (3),

$d = g\epsilon_r\epsilon_0$ =Charge sensitivity

1.7. OPTO-ELECTRONIC TRANSDUCER

Optoelectronics is the study and application of electronic devices that source, detect and control light, usually considered a sub-field of photonics. In this context, light often includes invisible forms of radiation such as gamma rays, X-rays, ultraviolet and infrared, in addition to visible light. Optoelectronic devices are electrical-to-optical or optical-to-electrical transducers, or instruments that use such devices in their operation.

Photoelectric transducers are an integral part of the optoelectronic transducers. They are the most used transducers for detection of light and controlling various systems.

Photoelectric Transducers

It is a device which allows flow of current only when the light falls on the device. The magnitude of current varies with the intensity of the light.

Photoelectric transducers operate on the principle that when light strikes on the photosensitive surface the following may result:

- Electrons may flow-in photoemissive cell
- A voltage may be generated-photovoltaic cell
- A change of resistance may take place-Photoconductive cell

This kind of photoelectric transducers have application in many of the electronic devices.

- The photoemissive Transducers are based on the emission of electrons from the metal cathode when it is exposed to light or other radiation. These electrons are collected by the positively charged anode. So this kind of transducers are used in sound reproduction of picture film, counting device and turn on and off a circuit etc.
- Photovoltaic cell-This generates an output voltage proportional to intensity of light or radiation intensity. Extremely used as a source of power for space aircraft. Other than that it is also used in TV, calculators and automatic control system etc.
- Photoconductive cell-In this device the resistance of the semiconductor decreases when intensity of light increases. Such devices have applications in industrial and laboratory control applications. These are used for relay operation. When light falls on the semiconductor then the resistance reduces and the relay operates otherwise it has high resistance. Also used to switch transistors on or off.
- Optocoupler: It is device in which the transistor is switched on using the light from the LED. It transfers electrical signals between two isolated circuits by using light. It consists of opto-transistor which turns on when light falls on its base terminal.

CHAPTER-2

2.1 Cathode Ray Tube (CRT)

A Cathode Ray Tube is the main part of Cathode Ray Oscilloscope (CRO) with additional circuitry to operate the circuit.

Its main parts are:

(1) Electron gun assembly.
(2) Deflection pate assembly.
(3) Fluorescent Screen.
(4) Glass envelope.
(5) Base for making the connection.

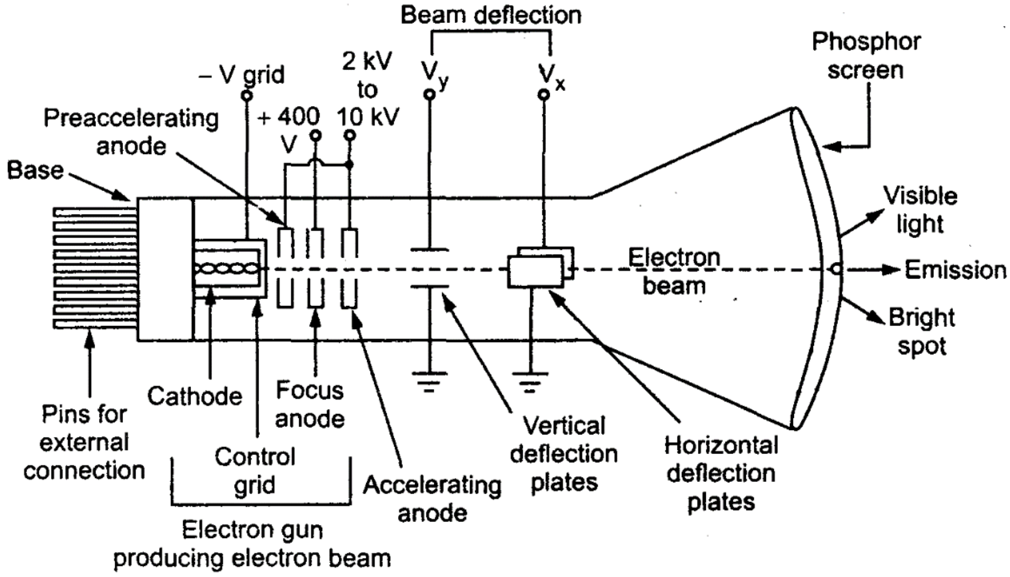


Fig. 2.1: Block Diagram of Cathode Ray Tube

Cathode ray tube is a vacuum tube and convert an electrical signal into visual one. Cathode ray tube makes available plenty of electrons. The electrons are accelerated to high velocity and are focussed on a fluorescent screen. Electron beam produces a spot of light wherever it strikes. The electron beam is deflected on its journey in response to the electrical signal under study. Hence the electrical signal is displayed.

Electron gun assembly

The arrangement of electron which produces a focussed beam of electrons is called the electron gun. It consists of:

(1) An indirectly heated cathode.
(2) A control grid surrounding the cathode.
(3) A focusing anode.
(4) An accelerating anode.

Control grid is held at negative potential with respect to cathode. Two anodes focusing anode and accelerating anode are maintained at high positive potential with respect to cathode.

It has the following parts:

- Cathode with the heater.
- Control grid
- Focusing anode
- Accelerating anode.

Cathode: It consists of a nickel cylinder coated with oxide coating of barium and strontium emit plenty of electron. Rate of emission of electrons depend upon magnitude cathode current which can be controlled by the control grid.

Control Grid: It is a nickel cylinder with a centrally located hole. It encloses the cathode and consists of a metal cylinder with a tiny circular opening to keep the electron beam small in size. It is 15mm diameter and 15mm long with a hole of 0.25mm at the centre. Control grid is kept at negative potential with respect to the cathode and its function is to vary the electron emission. The hole in the grid is provided to allow the passage of electrons and concentrate the beam of electron along the axis of the tube.

Focusing anode: The focusing anode focuses the electron beam into sharp pin point by controlling the positive potential on it. Its value is a few hundred volts and more positive than cathode so as to accelerate the electron beam.

Accelerating anode: The positive potential on the accelerating anode is much higher than the focusing anode. It is in the range of 10KV and for this reason the anode accelerate the narrow beam of electrons to high velocity. Therefore the electron gun assembly forms a narrow accelerated beam of electron s which produces a spot of light when strikes the screen.

Deflection Plate assembly

Deflection plate is accomplished by two set of deflecting plates placed within the tube beyond accelerating anode.

(1) One set is vertical deflection plates.
(2) Other set is horizontal defection plates.

Vertical defection plates:

These plates are mounted horizontally and applied a proper potential. These plates move the electron beam in the vertical direction when the potential is applied to the plates. These plates are responsible for the movement of electron beams in the vertical up and down on the fluorescent screen.

Horizontal defection plates:

These plates are mounted vertically and applied a proper potential. These plates move the electron beam in the horizontal direction when the potential is applied to the plates. These plates are responsible for the movement of electron beams in the horizontalleft and right on the fluorescent screen.

Fluorescent Screen:

The end wall or inside face of tube coated with some fluorescent material called:

- Phosphor
- Zinc oxide.
- Zinc orthosilicate

When high velocity of electron beam strikes the screen a spot of light is produced at the point of impact. It absorbs the kinetic energy of the electron and convert into light. Colour of the light emitted depends on the fluorescent material used.

Glass Envelope

It is a highly evacuated glass housing in which vacuum is maintained inside. Inner wall of the CRT between neck and the screen are usually coated with conducting material called “aquadag”. This coating is electrically connected to the accelerating anode so that electrons which accidentally stroke the wall return to the anode. This prevents the wall from charging to a high negative potential. This aquadag coating prevents the formation of negative charge on the screen.

2.2 Cathode Ray Oscilloscope (CRO)

Cathode ray oscilloscope is the most useful and the most versatile laboratory instrument for studying wave shapes of alternating current, voltage, power, frequency which have amplitude and waveform.

It allows the user to see the amplitude of electrical signal as a function of time on the screen.

With CRO wave shapes of signal can be studied and can be used for measuring

(1) Voltage
(2) Frequency
(3) Phase shift

Application of CRO

In laboratories it is used for

- Tracing actual waveform of current and voltage.
- Determination of amplitude of a variable quantity.
- Comparison of phase and frequency.
- Can be used to check any kind of electronic components in an electronic circuit.
- For finding B-H curve of hysteresis loop.
- For tracing the characteristics of any electronic components like diode, transistors etc.

For commercial purpose:

- In CRT televisions.
- In Radar.

Block diagram of General purpose CRO

It consists of the following components:

- Cathode Ray Tube
- Power supply Block
- Vertical Amplifier
- Horizontal Amplifier.
- Time Base generator.
- Trigger Circuit.

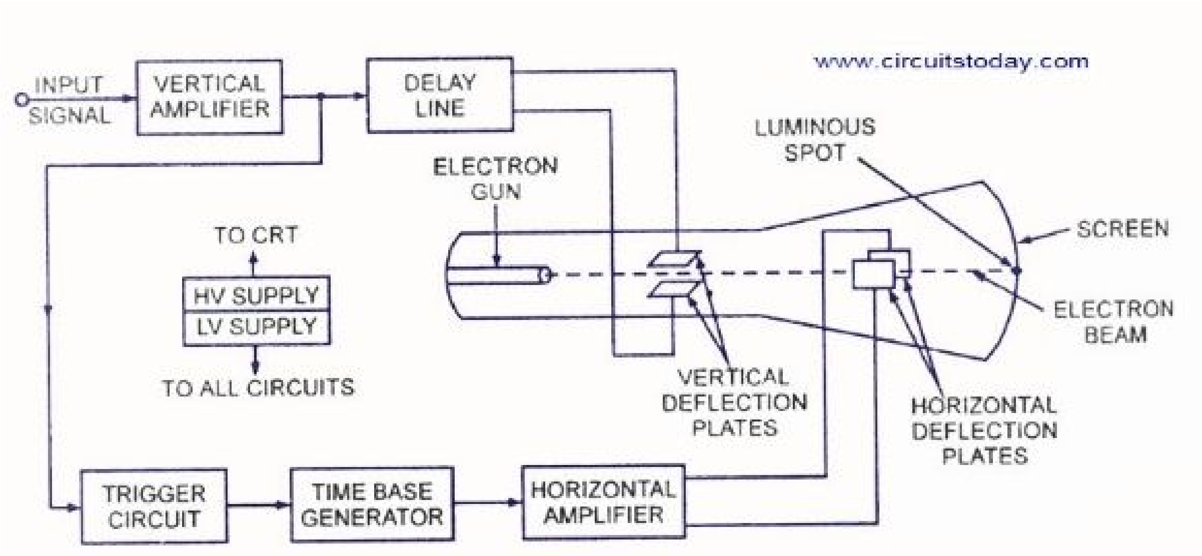


Fig 2.2: Block Diagram of Cathode Ray Oscilloscope

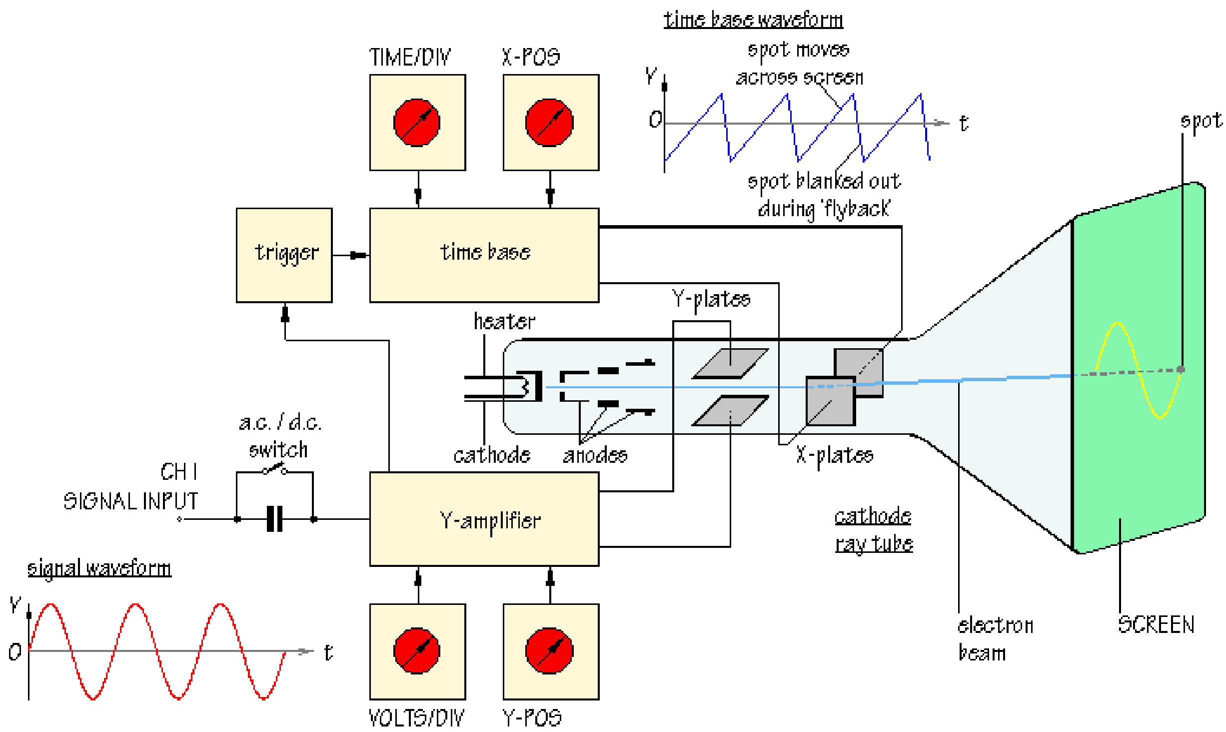


Fig. 2.3: Block Diagram of Cathode Ray Oscilloscope in a more practical way

Cathode Ray Tube:

It is the heart of the oscilloscope. It generates sharply focused electron beam and accelerate the beam to a high velocity and deflect the beam to create image. It contains the phosphor screen where the electron beam became visible. While travelling form the electron gun to screen the electron beam passes between set of

(1) Vertical Deflection Plate.

(2) Horizontal Deflection Plate.

Voltage applied to the vertical deflection plate to move the beam in the vertical plane and the CRT spot moves up and down.

Voltage applied to the horizontal deflection plate moves the beam in the horizontal plane.

Power supply block

It provides the voltage required by the CRT to generate and accelerate the electron beam as well as to supply the required operating voltages for the other circuit of the oscilloscope. Voltage in the range of few Kilo Volts are required by CRT for acceleration.

Low voltage for heater of electron gun of CRT which emit the electron.

Supply voltage for the other circuit is not more than few hundred volt.

Vertical Amplifier

It amplifies the signal waveform to be viewed. This is a wide bad amplifier used to amplify signal in the vertical section.

Horizontal Amplifier

It is fed with saw-tooth voltage. It amplifies saw-tooth voltage before it is applied to horizontal deflection plates.

Time base generator:

It develops saw-tooth voltage waveform required to deflect the beam in horizontal section.

Trigger Circuit

The trigger circuit is used to convert the incoming signal into trigger pulses so that input frequency can be synchronised.

2.3. CRO Measurement

Various parameters which can be measured by CRO are

1. Voltage; 2. Current; 3. Time period; 4. Frequency; 5.Phase angle

6. Amplitude, 7. Peak to peak value

Voltage measurement

Voltage to be measured is applied to the V deflection plates through vertical amplifier.

The ‘X’ deflection plate is excited by time base generator.

⇒ After noting down the selection in vect/div from front panel the peak to peak value amplitude and r m s value of sinusoidal voltage can be obtained.

I. Peak to peak value $V_{P-P} = \frac{Volt}{Division} \times no.of\ division$

II. Amplitude $V_m = \frac{V_{P-P}}{2}$

III. R. M. S value $V_{RMS} = \frac{2V_{P-P}}{\sqrt{2}} = \frac{V_P}{\sqrt{2}} + \frac{V_P}{\sqrt{2}}$

Voltage/division = deflection Sensitivity

Deflection factor $= \frac{1}{Deflection\ Sensitivity}$

Current Measurement

A CRO has a very high input impedance and cannot be used for direct measurement. However the current can be measured in terms of voltage drop across a standard resistance $I = \frac{V}{R}$

Time period measurement:

For the measurement of time period ‘T’ of the waveform. It is displayed on the screen such that one complete cycle is visible on the screen.

After noting the time/division selected on front panel the time period of the wave form can be obtained as Time period T = time/division X number of division occupied by one cycle.

Frequency = f = 1/T

Frequency measurement by (lisajous Method Pattern)

The unknown frequency can be accurately determined with the help of CRO

Step – 1

Known frequency is applied to horizontal input (2000 hz) Unknown frequency to vertical input

The number of loops cut by the horizontal line gives frequency on the vertical plates fy

The number of loops cut by vertical lines gives the frequency on the horizontal plates F_H

$$\frac{f_v}{f_H} = \frac{\text{No of loops cut by horizontal line}}{\text{No of loops cut by vertical line}}$$

$$\frac{f_v}{2000} = \frac{1}{2}$$

$$f_v = 2000 \times \frac{1}{2}$$

Unknown frequency = 1000hz

It is a method to determine the unknown frequency by comparing it with known frequency.

This fog = Lissajous fog – Named after French.

Example 1:

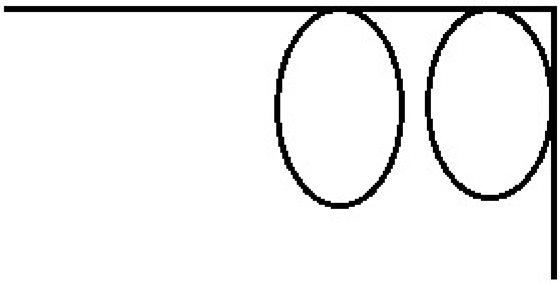
$$\frac{f_v}{f_H} = \frac{1}{1}$$

f_H= 1000

f_v= f_H= 1000hz

No of loops cut by horizontal and vertical line – 1

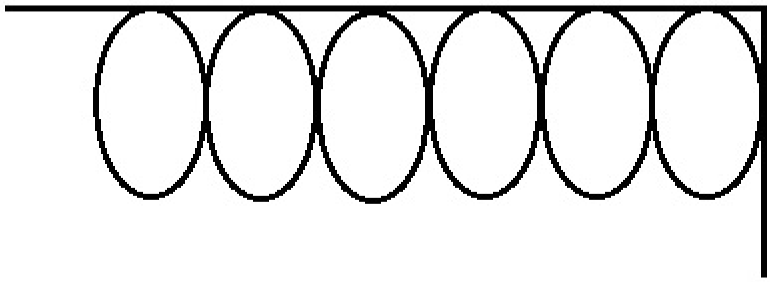
Example 2:



Noi of loops cut by horizontal line = 2

f_v/ f_H 2/1, $f_v/1000 = 2/1$ f_v= 2000hz

Example 3:



No of cut by horizontal
line = 6
vertical = 1
f_v/ f_H= 6/1 f_v/ 1000 = 6/1 f_v = 6 X 1000 = 6000 hz.

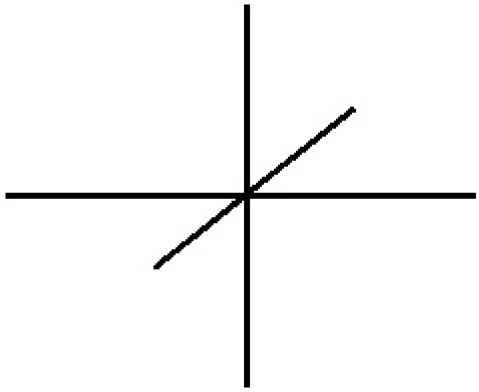
2.4. Phase and frequency measurement

An as well as cope can be used to find the phase angle between two sinusoidal quantities of the same frequency.

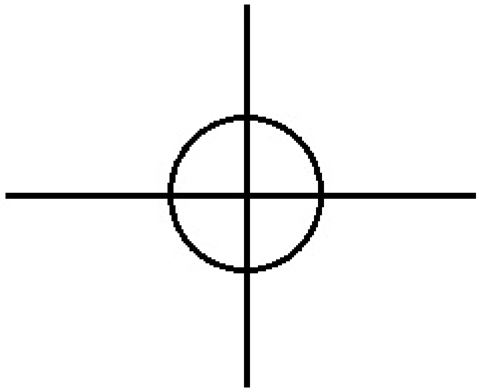
I. One of the signals is applied to Y plates time base generator is switched out and second signal is fed to X plates.

It is necessary that X^2 Y are equal magnitude.

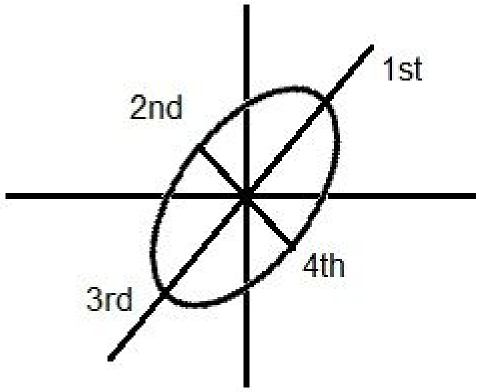
- If the two signals are in phase the display would be a straight line at 45° to the horizontal.



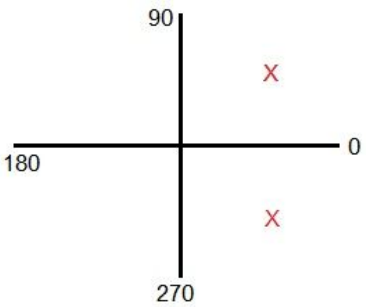
- If the phase angle is 90˚ the display would be a **circle.**

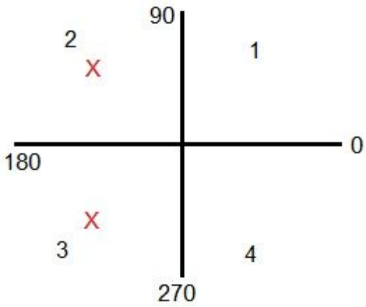


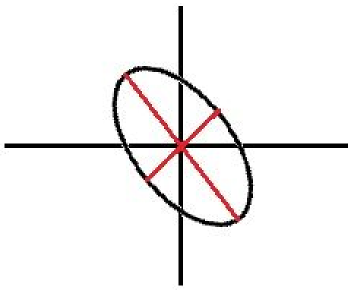
- For any other phase difference the display would be an ellipse.



If the phase angle between (0 - 90˚) = (270˚ - 360˚)
The ellipse has its major axis in the **1st** quadrant and **3rd** quadrant.







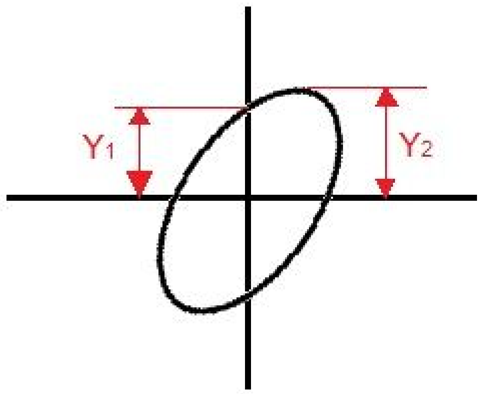
If the phase difference is between 90˚ and 180˚ the ellipse has its major axis in the 2nd and 4th quadrant.

Value of phase angle ø is given by

$\sin ø = Y_1/Y_2$ are intercept.

I When the phase angle is between **<u>0 to 90˚</u>**

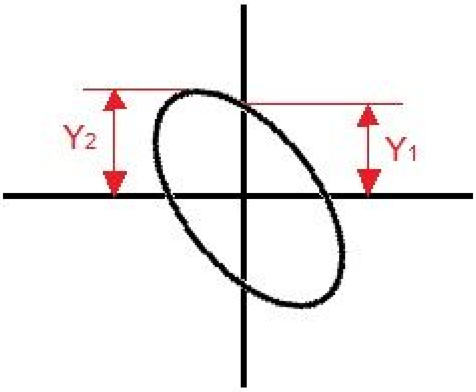
& 270 to 360˚ major axes lines in 1st quadrant = 3rd quadrant



$\sin ø = Y_1/Y_2 = ½ = 0.5$

Ø= sin -1 0.5 = 30˚ or **+5**

II. When phase angle is between 90˚ to 180˚ and 180˚ to 270˚ major axis would lie in 2nd and 4th quadrant.



$\sin ø = Y_1/Y_2 = 0.5$
$Ø = 150^\circ$ or 210°

CHAPTER-3

Measurement of pressure by Pirani Gauges

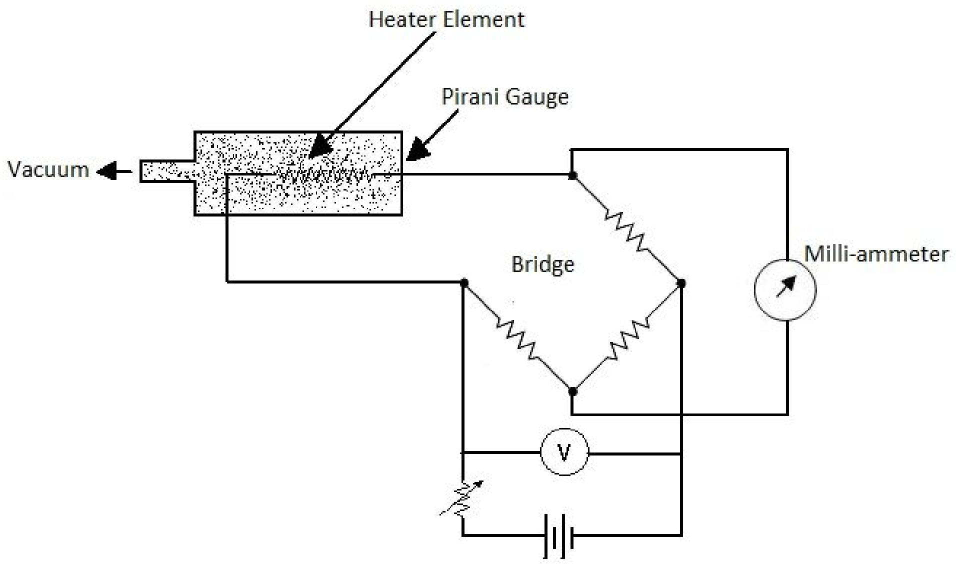


Fig. 3.1: Schematic diagram of Pirani Gauge

The operation of Pirani Gauges depend on variation of the thermal conductivity of a gas with pressure.

Thermal conductivity of the gas is measured by detecting the amount of heat oss form an electrically heated wire placed in the gas. Heat is conducted from wire by conduction through the gas and greater the thermal conductivity of the gas, the lower will be the temperature of the heater wire. Since, resistance varies with temperature, the resistance of the heater wire is the measure of pressure.

A Wheatstone bridge is used to measure the resistance of heater wire.

Advantages:

- It is useful for the pressures ranging from $10^{-1}$to 10^{-3} mm of mercury.
- These gauges are rugged, inexpensive and more accurate than thermocouple gauges.

Disadvantages:

- At very low pressure, the amount of heat conducted becomes very small and the method cannot be used.
- It must be frequently checked and calibrated.

Measurement of Pressure using Inductive Transducers

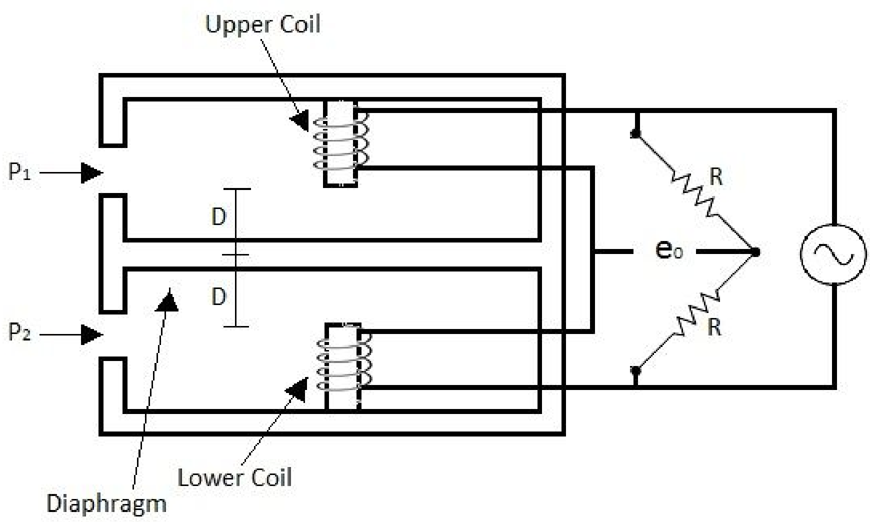


Fig. 3.2: Schematic diagram of Inductive Transducers

The arrangement uses two coils:

Upper coil.

Lower coil.

These are two arms of an AC bridge. The coils have equal number of turns. The other two arms consists two equal resistances of value R. The diaphragm is symmetrically placed with respect to the coils.

When $P_1 = P_2$,

Reluctances of the paths of magnetic flux for both coils are equal and hence inductance are equal.

$$\text{Self-inductance} = \frac{N^2}{R_0}$$

N = Number of turns.

R_0 = Initial reluctance of flux path

Under this condition, bridge is balanced and output $e_0 = 0$.

When $P_1 > P_2$,

Differential pressure, $P = P_2 - P_1$

Hence it deflects the diaphragm upwards through a distance d.

For small displacement of diaphragm, the reluctance of the flux path of the of the upper coil is $R_1 = R_0 + k(D - d)$ and the lower coil is $R_2 = R_0 + k(D + d)$

Therefore, Inductance of upper coil,

$$L_1 = \frac{N^2}{R_1}$$

$$= \frac{N^2}{R_0 + k(D - d)}$$

Inductance of lower coil,

$$L_2 = \frac{N^2}{R_2}$$

$$= \frac{N^2}{R_0 + k(D + d)}$$

When the bridge becomes unbalanced, the output voltage is given by,

$$e_0 = \left[\frac{1}{2} - \frac{L_2}{L_1 + L_2}\right] e_i$$

$$= \left[\frac{1}{2} - \frac{\frac{N^2}{R_0 + k(D+d)}}{\frac{N^2}{R_0 + k(D-d)} + \frac{N^2}{R_0 + k(D+d)}}\right] e_i$$

$$= \frac{K_d . e_i}{2(R_0 + KD)}$$

K, R_0, D, e_i are constant. Therefore output voltage directly proportional to d.

Measurement of pressure using capacitive transducer

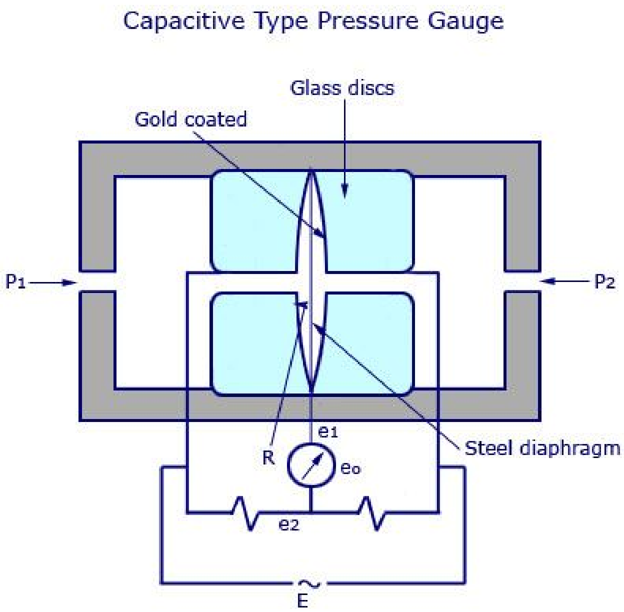


Fig. 3.3: Schematic diagram of Capacitive Transducer

- Capacitive transducer are used for measurement of pressure by converting the pressure into a displacement. The displacement is sensed by capacitive transducer using a differential arrangement.
- The use of a three terminal variable differential capacitor is used for measurement of different pressure with the help of diaphragm.
- The spherical depressions are stationary into the glass discs and are coated with gold to form the two fixed plates of the differential capacitor. The thin diaphragm is clamped between these two plates.
- When $P_1 = P_2$

Diaphragm is in neutral position and the bridge is balanced and the output voltage $e_0 = 0$.

- When $P_1 > P_2$ or $P_1 < P_2$,
  The diaphragm deflects giving output voltage e_0.
- This voltage may be amplified by an emitter follower amplifier.
- Disadvantages:
  - Low sensitivity.
  - Requirement of high frequency (2500 Hz).

Measurement of temperatures by thermocouple

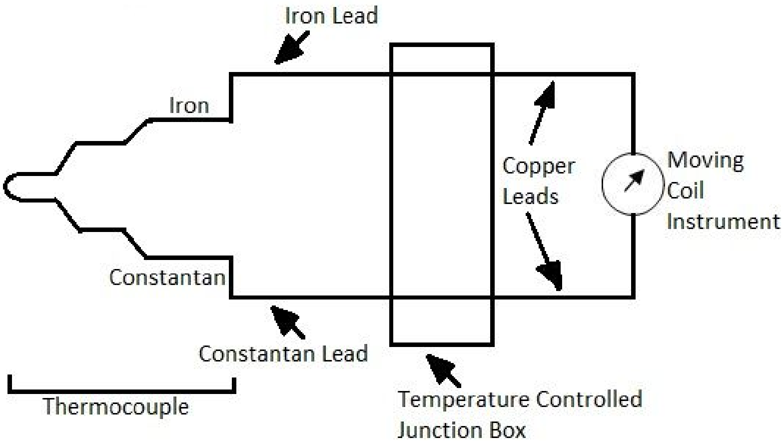


Fig. 3.4: Schematic diagram of Thermocouple

The EMF produced on a thermocouple circuit is given by:

$E = a\ (\Delta\theta) + b\ (\Delta\theta)^2$

$\Delta\theta =$ Difference in temperature between hot thermocouple junction and reference junction of the thermocouple $(deg\ C)$

$a, b =$ Constants

Usually,

$a \gg b$

Therefore,

$E = a\ (\Delta\theta)$

Or, $$\Delta\theta = \frac{E}{a}$$

The EMF setup is measured by sending a current through a moving coil instruments, the deflection being directly proportional to the EMF. Since EMF is a function of temperature difference $\Delta\theta$, the instrument can be calibrated to read temperature.

The temperature of the cold junction i.e. reference junction should remain absolutely constant in order that the calibration is good.

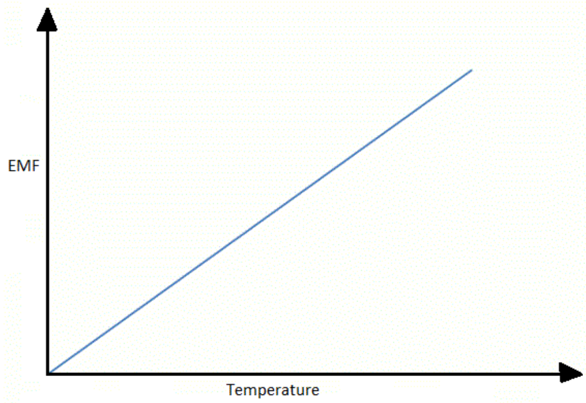


Fig. 3.4: EMF temperature variation of a thermocouple

Thermocouples can measure upto 1400^0C.

Measurement of temperatures by Pyrometers

Pyrometers

When the temperatures under measurement are high and physical contact in the process is impractical, then thermal radiations measures are used. This is done by optical pyrometers. Pyrometers are used under a condition when the corrosive vapours or liquids will destroy the thermocouples and the range of temperature is to be measured is very high that the thermocouple cannot withstand.

Radiation pyrometers measures the heat emitted or reflected by a hot object. Thermal radiation is electromagnetic radiation emitted as a result of temperature.

Optical Pyrometers

The radiation from a heated body with high temperatures, fall within the visible region of the electromagnetic spectrum. For a given wavelength in the visible region the energy radiated is greater at higher temperatures. Within the visible region the given wavelength has a fixed colour and the energy of the radiations is interpreted as intensity or brightness. Thus, the brightness of the light of a given colour emitted by the hot body gives us the indication of temperature of that body. This is the principle of working of an optical pyrometers.

Disappearing filament optical pyrometers

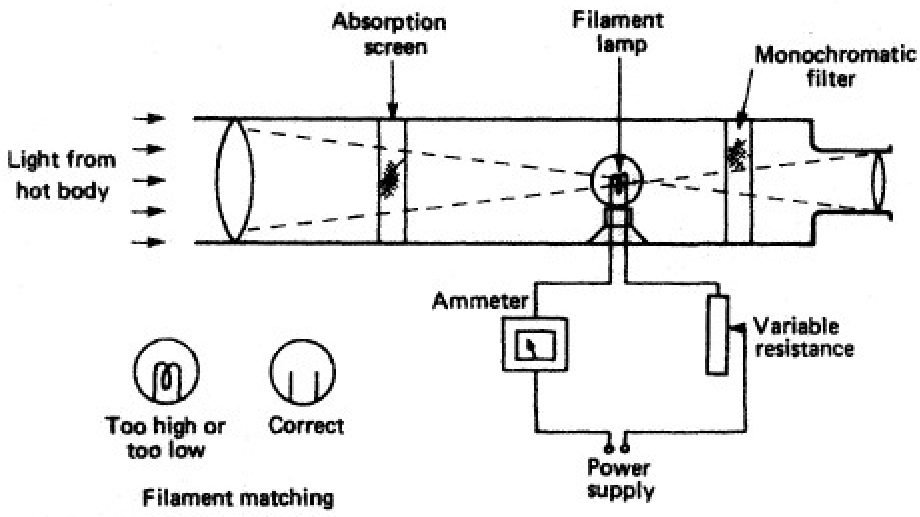


Fig. 3.5: Schematic diagram and working of optical pyrometers

- Radiant energy is produced by a heated body.
- Radiant energy first falls on the lamp filament through the objective lens and absorption screen.
- Current through the filament is adjusted so that the lamp intensity can be adjusted. This adjustment is made till the filament and the image are of equal brightness.
- When the brightness of the filament is equal to the brightness of the image then the outline of the filament disappears.
- The PMMC meter can be directly calibrated to give the brightness in terms of filament current.
- The range of the temperature that can be measured using this is 1400^0C.

Measurement of flow using turbine meters

- Turbine flow meters are volumetric flow meters and are available in wide ranges.
- The output is in the form of digital electrical signals whose frequency is directlyproportional to flow rate and where total count is proportional to total quantity, as each pulse represents a discrete volume.

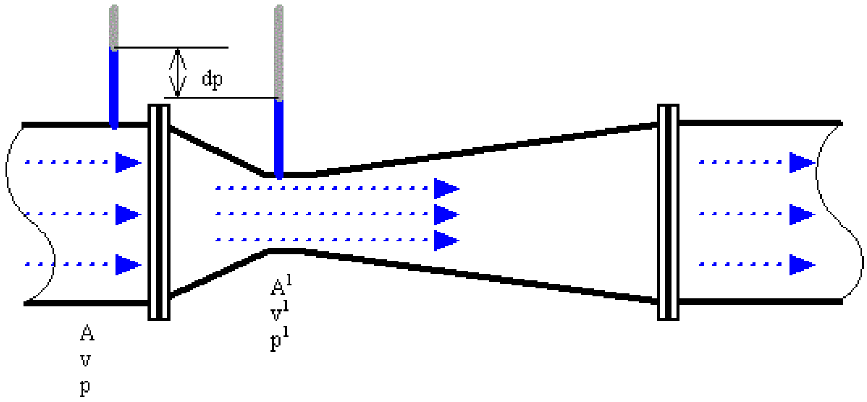


Fig. 3.6: Schematic diagram Venturi flow meter

Advantage

- The output is in electrical form which can be used at distance for recording and control.
- Passage of rotor past the pick-up coil produces as EMF which can be converted to DC analog voltage by D/A converters.
- Error is approximately $\pm$0.5%.

Drawbacks

- Error may be caused due to excessive functional torques, which may be aroused due to wear and tear of bearings.
- Variation in performance due to changes in the characteristics of liquid, friction and bearing etc.

CONTROL SYSTEM

CHAPTER- 4

4.1 INTRODUCTION

A system is a collection of objects(Components) connected together to serve an objective, or a system is a combination of components that act together to perform an objective. The control system is that means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner.

A physical system is a collection of physical objects connected together to serve an objective. No physical system can be represented in its full physical intricacies, and therefore, idealizing assumptions are always made for the purpose of analysis and synthesis of systems. An idealized physical system is called a physical model. A physical system can be modelled in a number of ways depending upon the specific problem to be dealt with and the desired accuracy. Once a physical model of a physical system is obtained, the next step is to obtain a mathematical model, which is the mathematical representation of the physical model, through use of appropriate physical laws. Depending on the choice of variables and the coordinate system, a given physical model may lead to different mathematical models. A control system may be modelled as a scalar differential equation describing the system or as a state variable vector-matrix differential equation. The particular mathematical model which gives a greater insight into the dynamic behaviour of the physical system is selected. When the mathematical model of a physical system is solved for various input conditions, the result represents the dynamic response of the system.

4.2 CLASSIFICATION OF CONTROL SYSTEM

Control systems may be classified in a number of ways depending on the purpose of classification.

1. Depending on the hierarchy, control systems may be classified as

(a) Open-loop control systems
(b) Closed-loop control systems
(c) Optimal control systems
(d) Adaptive control systems
(e) Learning control systems

2. Depending on the presence of human being as a part of control system, control systems may be classified as

(a) Manually controlled systems

(b) Automatic control systems

3. Depending on the presence of feedback, control systems may be classified as

(a) Open-loop control systems

(b) Closed-loop control systems or feedback control systems

4. According to the main purpose of the system, control systems may be classified as

(a) Position control systems

(b) Velocity control systems

(c) Process control systems

(d) Temperature control systems

(e) Traffic control systems etc.

Feedback control systems may be classified in a number of ways depending on the purpose of classification.

1. According to the method of analysis and design, control systems may be classified as linear control systems and nonlinear control systems

2. Depending on whether the parameters of the system remain constant or vary with time, control systems may be classified as time-varying or time-invariant control systems.

3. According to the type of signals used in the system, control systems may be classified as

(a) continuous – data control systems and discrete-data control systems

(b) ac(modulated) control systems and dc(unmodulated) control systems

4. Depending on the application ,control systems may be classified as position control systems, velocity control systems, process control systems, traffic control systems etc.

5. Depending on the number of inputs and outputs, control systems may be classified as single-input-single-output(SISO) control systems and multi-input-multi-output (MIMO) control systems.

6. Depending on the number of open-loop poles of the system transfer function present at the origin of s-plane, control systems may be classified as

(a) Type-0

(b) Type-1

(c) Type-2 etc.

7. Depending on the order of the differential equation used to describe the system, control systems may be classified as first-order control systems, second-order control systems etc.

8. Depending on the type of damping, control systems may be classified as

(a) Un-damped systems

(b) Under-damped systems

(c) Critically damped systems

(d) Over-damped systems

4.3 & 4.4 OPEN-LOOP AND CLOSED LOOP SYSTEMS

Consider for example the driving system of an automobile. Speed of the automobile is a function of its accelerator. The desired speed can be maintained (or a desired change in speed can be achieved) by controlling pressure on the accelerator pedal. This automobile driving system (accelerator, carburettor & engine vehicle) constitutes the control system.fig.4.1 shows the general diagrammatic representation of a typical control system. For the automobile driving system the (liquid form) flow to the engine bringing the engine- vehicle speed (controlled variable) to the desired value.

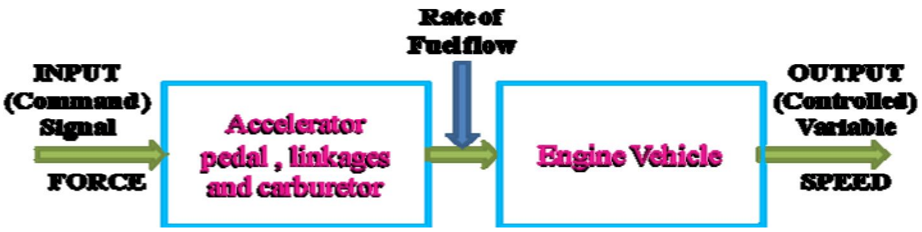


Fig.4.1 The basic control system

The diagrammatic representation of Fig.4.1 is known as block diagram representation where in each block represents an element, a plant, mechanism, etc. whose inner details are not indicated .Each block has an input and output signal which are linked by a relationship characterizing the block. It may be noted that the signal flow through the block is unidirectional.

Let us reconsider the above automobile driving system. The route, speed & acceleration and of the automobile are determined and controlled by the driver by observing traffic and road conditions are by properly manipulating the accelerator, clutch, gear-lever, breakers and steering wheel etc. Suppose the driver wants to maintain a speed of 50Km per hour(desired output).he accelerates the automobile to this speed with the help of the accelerator and then maintains it by holding the accelerator steady. No error in the speed of the automobile occurs so long as there are no gradients or other disturbances along the road .The actual speed of the automobile is measured by a speedometer and indicated on its dial. The driver reads the speed dial visually and compares the actual speed with the desired one mentally. If there is a deviation of speed from the desired speed ,accordingly he takes the decision to increase or decrease the speed. The decision is executed by the change in pressure of his foot (through muscular power)on the accelerator pedal.

These operations can be represented in a diagrammatic form as shown in Fig.4.2. In contrast to the sequence of events in Fig.4.1,the events in the control sequence Fig.4.2 follow a closed- loop i.e. the information about the instantaneous state of the output is feedback to the input and is used to modify it in such a manner as such to achieve the desired output. It is on account of this basic difference that the system of Fig.4.1 is called an **open-loop-system**, while the system of Fig.4.2. is called a **closed-loop-system.**

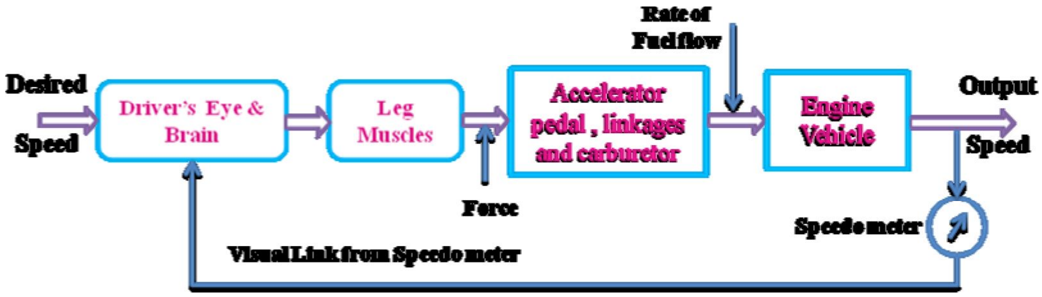


Fig. 4.2 Schematic diagram of a manually controlled closed-loop system

Let us investigate another control aspect of the above e.g. of an automobile (engine vehicle) say its steering mechanism. A simple block diagram of an automobile steering mechanism is shown in Fig.4.3 (a).The driver senses visually and tactile means (body movement) the error between the actual and desired directions of the automobile as in Fig.4.3 (b). Additional information is available to the driver form the feel(sensing)of the steering wheel through his hands,theses information constitute the feedback signal(s) which are interpreted by driver's brain who then signals his hand to adjust the signal accordingly. This again an e.g. of a close-loop system where human visual and tactile measurements constitute the feedback loop

In fact unless human being(s) are not left out of in a control system study practically all control system are a sort of closed-loop system (with intelligent measurement and sensing loop or there may indeed by several such loops \).

System of the type represented in Fig.4.2 and 4.3 involve continuous manual control by a human operator. These are classified as **manually controlled systems.** In many fast-acting systems, the presence of human element in the control loop is undesirable because the system response may be too rapid for an operator to follow or the demand on operator's skill maybe unreasonably high. Furthermore some like or missile propelling etc. are self-destructive & in such systems human element must be excluded. Even in situations where manual control could be possible an economic case can often be made out for reduction of human supervision. Thus in most situations the use of some equipment performs the same intended function as a continuously employed human operator is preferred. A system incorporating such equipment is known as automatic control system in automatic control system. In fact in most situations an automatic control system could be made to perform intended functions better that a human operator, and could further be made to perform such functions as would be impossible for a human operator.

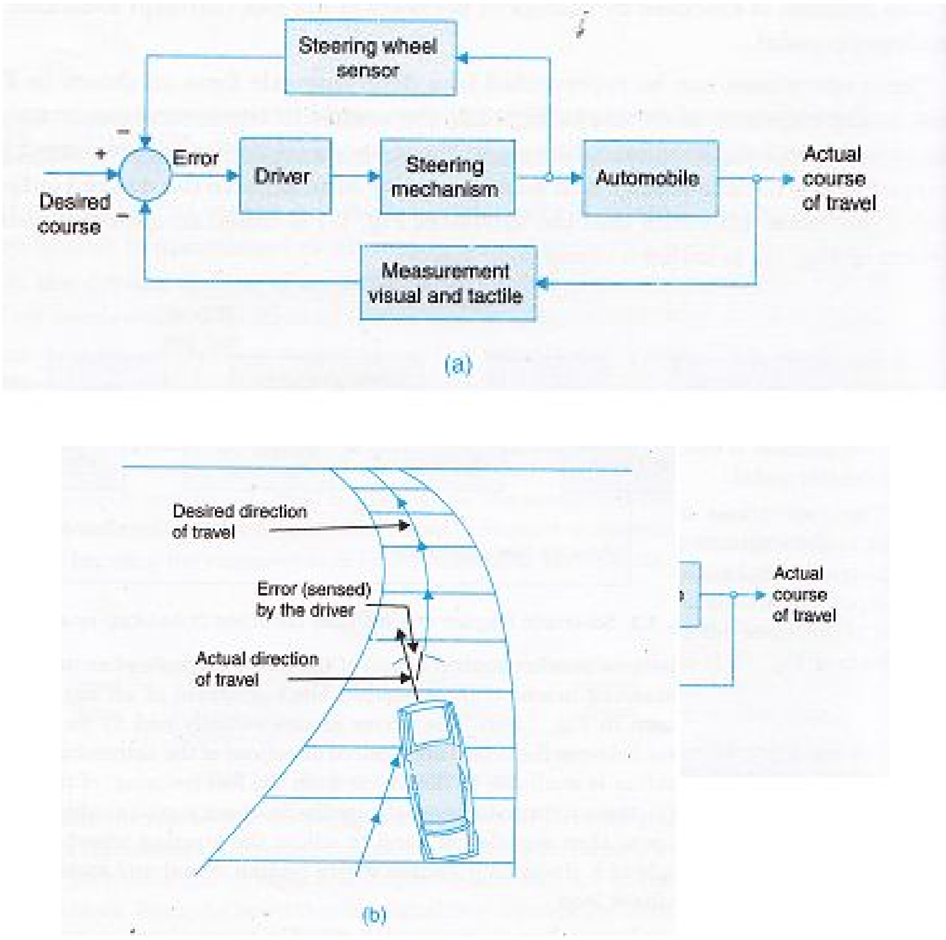


Fig 4.3(a) Automobile steering control system

(b) The driver uses the difference between the actual and desired direction of the travel to adjust the steering wheel accordingly

The general block diagram of an automatic control system which is characterized by a feedback loop, is shown in Fig 4.4, an error detector compares a signal obtained through feedback elements, which is a function of the output response, with the reference input. Any difference between these two signals constitutes an error or actuating signal, which actuates the control elements. The control elements in turn actuate the plant (controlled member) in such a manner as to reduce the original error.

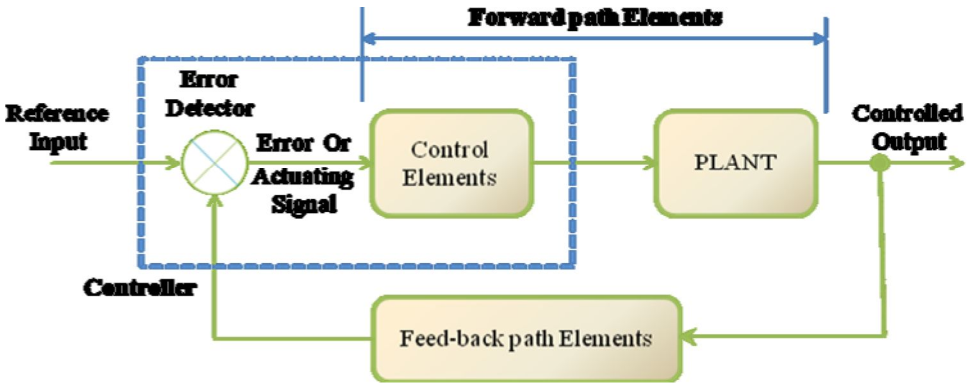


Figure 4.4. Error detector comparing the reference input & controlled output

4.5 **Comparison of Open-loop control and closed loop control system**

As it is known any physical system which does not automatically correct variations in its output, is called an open-loop system. Such a system may be represented by the block diagram 4.5.In these systems the output remains constant for a constant input signal provided the external conditions remain unaltered. The output may be changed to any desired value by appropriately changing the input signal but variations in external conditions or internal parameters of the system may cause the output to vary from the desired value in an uncontrolled fashion. The open-loop control is, therefore, satisfactory only if such fluctuations can be tolerated or system components are designed and constructed so as to limit parameter variations and environmental conditions are well controlled.

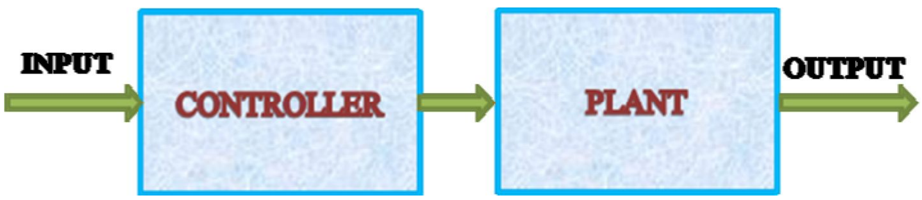


Fig 4.5. An open-loop system which does not automatically correct the output

It is important to note that the fundamental difference between an open loop and close-loop control system is that of feedback action. Consider, for example, a traffic control system for regulating the flow of traffic at the crossing of two roads. The system will be termed open-loop if red and green lights are put on by a timer mechanism for predetermined fixed intervals of time. It is obvious that such an arrangement takes no account of varying rates of traffic flowing to the road crossing from two directions. If on the other hand, a scheme is introduced in which the rates of traffic flow along both directions are measured and are compared and the difference is used to control the timing of red and green lights, a closed-loop system (feedback system) results. Thus the concept of feedback can be usefully employed to traffic control.

Advantages of open loop control

1. Open loop control is much simpler and less expensive.
2. No sensor is needed to measure the variables to provide feedback.

Disadvantages of open loop control

1. No accuracy.
2. There is no compensation for any disturbance entering into the system since input is independent of disturbance.
3. Its performance highly depends on the properties of the system which sometimes may vary with respect to time.

Advantages of closed- loop control

1. It can be designed to provide extreme accuracy at a steady state.
2. Its time response can be adjusted by appropriate design.
3. It compensates for disturbances.
4. Transient response and steady state response can be controlled more conveniently
5. and with great flexibility.
6. Less sensitive to system parameters variation.

Disadvantages of closed loop control

1. More complex and expensive.
2. Reduce the gain of the system.
3. If the closed loop system is not properly designed, the feedback system may lead to undesirable response.

4.6 FEEDBACK AND ITS EFFECTS

Feedback systems play an important role in modern engineering practice because they have the possibility for being adopted to perform their assigned tasks automatically. A non-feedback (open-loop) system represented by the block diagram is shown in Fig.4.6(a),is activated by a single signal at the input(for single input systems).There is no provision within this system for supervision of the output and no mechanism is provided correct(or compensate) the system behaviour for any lack of proper performance of system components, changing environment loading or ignorance of the exact value of process parameters. On the other hand, a feedback (closed loop) system represented by the block diagram as shown in Fig.4.6 (b) is driven by two signals, one the input signal and the other, a signal called the feedback signal derived from the output of the system. The feedback signal gives the system the capability to act as self-correcting mechanism as explained below.

The output signal c is measured by a sensor H(s), which produces a feedback signal b. The comparator compares the feedback signal b with the input(command) signal r generating the actuating signal e, which is as measure of discrepancy between r and b

.the actuating signal is applied to the process G(s) so as to influence the output c in a manner which tends to reduce the errors.

Feedback as a means of automatic regulation and control is, in fact, inherent in nature and can be noticed in many physical, biological and soft systems. For example, the body temperature of any living being is automatically regulated through a process which is essentially a feedback process, only it is far more complex than the diagram of Fig.4.6 (b).

The beneficial effects of feedback in feedback systems with high loop gain are enumerated below.

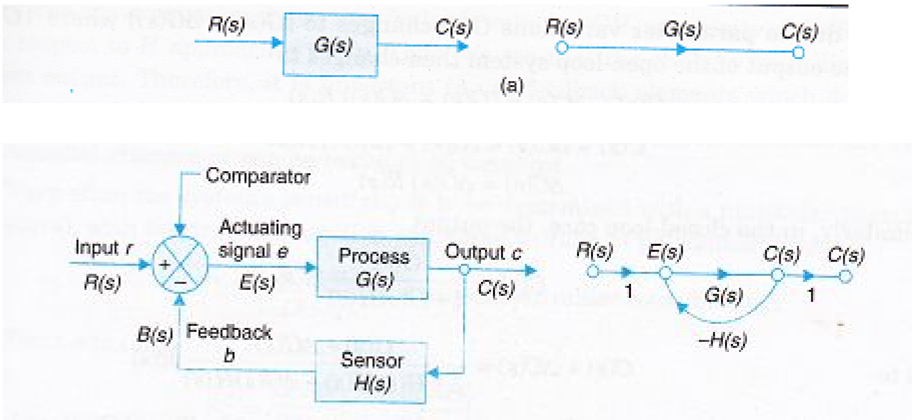


Fig 4.6.(a) A non-feedback (open-loop) system, (b) A feedback (close-loop) system

1. The controlled variable accurately follows the desired value.
2. Effect on the controlled variable of external disturbances (other than, those associated with the feedback sensor) is greatly reduced.
3. Effects of variation in controller and process parameters (the forward path) on system performance are reduced to acceptable levels. These variations occur due to wear, aging, environmental changes etc.
4. Feedback in the control loop allows accurate control of the output (by means of the input signal) even when process or controlled plant parameters are not known accurately.
5. Feedback in a control system greatly improves the speed of its response compared to the response speed capability of the plant/components composing the system (forward path).

EFFECT OF FEEDBACK

1. REDUCTION OF PARAMETER VARIATIONS BY USE OF FEEDBACK

One of the primary purpose of using feedback in control systems is to reduce the sensitivity of the system to parameter variations. The parameters of a system may vary with age, with changing environment etc. Conceptually, sensitivity is a measure of the effectiveness of feedback in reducing the influence of these variations on system performance.

Let us define sensitivity on a quantitative basis. In the open-loop case

$C(s) = G(s)\,R(s)$

Suppose due to parameter variations G(s) changes to $[G(s) + \Delta G(s)]$ where $|G(s)| >> |\Delta G(s)|$. The output of the open-loop system then changes to

$C(s)+ \Delta C(s) = [G(s)+ \Delta G(s)]R(s)$

Or $\Delta C(s) = \Delta G(s)\,R(s)$ 4.1

Similarly, in the closed-loop case, the output

$$C(s) = \frac{G(s)}{1+G(s)H(s)}\,R(s)$$

Changes to $C(s) + \Delta C(s) = \frac{G(s)+\Delta G(s)}{1+G(s)H(s)+\Delta G(s)H(s)}\,R(s)$

Due to the variations of $\Delta G(s)$ in G(s) & as $|G(s)| >> |\Delta G(s)|$ we have the variation in the output as

$$\Delta C(s) = \frac{\Delta G(s)}{1+G(s)H(s)}\,R(s) \quad ....4.2$$

From the equations 4.1 and 4.2 it is seen that in comparison to the open-loop system, the change in output of the close-loop system due to variation G(s) is reduced by a factor of [1+G(s)H(s)] which is much greater than unity in most practical cases over the frequency of interest.

The term system sensitivity is used to describe the relative variation in the overall transfer function T(s) = C(s)/R(s) due to variation in G(s) and is defined below.

Sensitivity = Percentage change in T(s)/ Percentage change in G(s)

EFFECT OF FEEDBACK ON OVERALL GAIN

As feedback affects the gain G(s) of a non feedback system by a factor $\frac{1}{1+G(s)H(s)}$ .The system can have a negative or positive feedback which is indicated at the comparator stage with sign accordingly. The quantity G(s)H(s) mat itself include a negative sign ,so the general effect of feedback is that it may increase or decrease the gain. In practical control systems ,G(s) and H(s) are function of frequency , so the magnitude [1+G(S)H(s)] may be greater than 1 in one frequency range but less than 1 in another. Therefore, feedback could increase the system gain in one frequency range but decrease in another.

Therefore, feedback could increase the system gain in one frequency range but decrease it in other.

EFFECT OF FEEDBACK ON STABILITY

Stability is a notion that describes whether the system will be able to follow the input command, or be used in general. In a non rigorous manner, a system is said to be unstable if its output is out of control. To investigate the effect of feedback on stability, for G(s)H(s) - -1,the output of the system is infinite for any finite input and the system is said to be unstable. Therefore, we may state that feedback can cause a system that is originally stable to become unstable .Certainly, feedback is a two-edged sword, when used improperly, and it can be harmful. In general, G(s) H(s) = -1 is not the only condition for instability.

EFFECT OF FEEDBACK ON EXTERNAL DISTURBANCE OR NOISE

All physical systems are subject to some types of extraneous signals or noise during operation. Examples of these signals are thermal noise voltage in electronic circuits and brush or commutator noise in electrical motors. External disturbances, for example wind gust acting on an antenna, are also quite common in control systems. Hence, in the design of control systems, consideration should be given so that the system is insensitive to noise and disturbances and sensitive to input commands.

The effect of feedback on noise and disturbance depends greatly on where these extra noise signals occur in a system. No general conclusion can be reached, but in many situations, feedback can reduce the effect of noise and disturbances on system performance.

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SERVOMECHANISMS

CHAPTER -5

5.1 INTRODUCTION

In modern usage the term servomechanism or servo is restricted to feedback control systems in which the controlled variable is mechanical position or time derivatives of position, e.g., velocity and acceleration.

5.2 AUTOMATIC TANK-LEVEL CONTROL SYSTEM

In order to gain a better understanding of the interactions of the constituents of a control system, let us discuss a simple tank level control system shown in Fig.5.1.

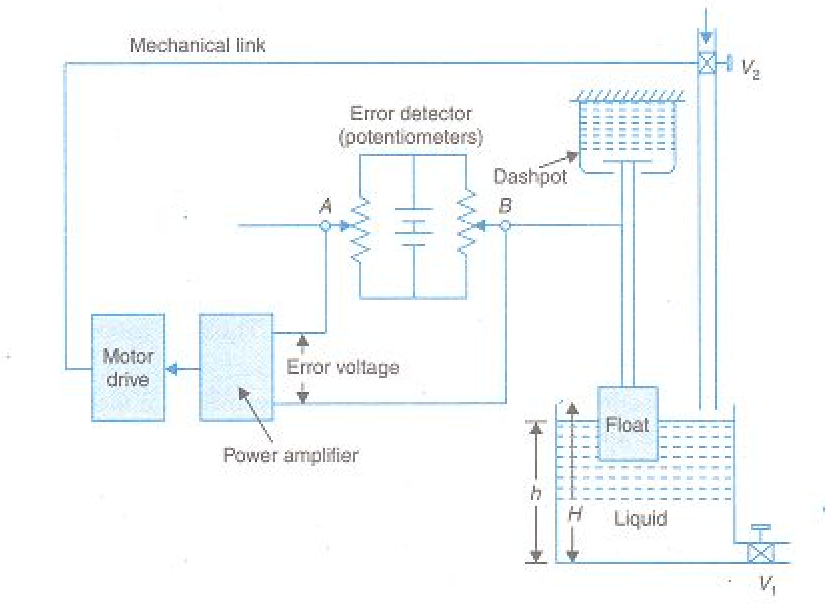


Fig 5.1. An automatic Tank Level Control System

This control system can maintain the liquid level h(controlled output) of the tank within accurate tolerance of the desired liquid level even though the output flow rate through the valve V_1 is varied. The liquid level is sensed by a float (feedback path element), which positions the slider arm B on a potentiometer. The slider arm A of another potentiometer is positioned corresponding to the desired liquid level H(the reference input).When the liquid level rise or falls ,the potentiometers(error detector) give an error voltage (error or actuating signal)proportional to the change in liquid level. The error voltage actuates the motor through a power amplifier (control elements) which in turn conditions the plant (i.e. decrease or increase the opening of the valve, V_2) in order to restore the desired liquid level. Thus the control system automatically attempts to correct any deviation between the actual and desired liquid levels in the tank.

5.3 POSITION CONTROL SYSTEM

A servo system used to position a load shaft is shown in fig.5.2 in which the driving motor is geared to the load to be moved. The output (controlled) and desired (reference) positions θ_c and θ_R respectively are measured and compared by a potentiometer pair whose output voltage v_E is proportional to the error in angular position $\theta_E = \theta_R - \theta_c$ .The voltage $v_E = K_p\theta_E$ is amplified and is used to control the field current (excitation of a dc generator which supplies the armature voltage to the drive motor.

To understand the operation of the system assume K_p = 100 volts/radian and let the output shaft position be 0.5 radian. Corresponding to this condition the slider arm B has a voltage of +50 volts. Let the slider arm A be also set at +50 volts .This gives zero actuating signal (v_E = 0).Thus the motor has zero output torque so that the load stays stationary at 0.5 radian.

Assume now that, the new desired load position is 0.6 radian. To achieve this, the arm A is placed at +60 volts position, while the arm B remains instantaneously at + 50 volts position. This creates an actuating signal of +10 volts, which is a measure of lack of correspondence between the actual load position and the commanded position. The actuating signal is amplified and fed to the servomotor which in turn generates an output torque which repositions the load. The system comes to a standstill only when the actuating signal becomes zero i.e., the arm B and the load is the position corresponding to 0.6 radian (+60 volts position)

Consider now that a load torque T_L is applied at the output as indicated in Figure1.7.this will require a steady value of error voltage v_E which acting through the amplifier ,generator ,motor and gears will counterbalance the load torque. This would mean that a steady error will exist between the input and output angles. This is unlike the case when there is no-load torque and consequently the angle error is zero. In control terminology, such loads are known as load disturbances and system has to be designed to keep the error to these disturbances within specified limits.

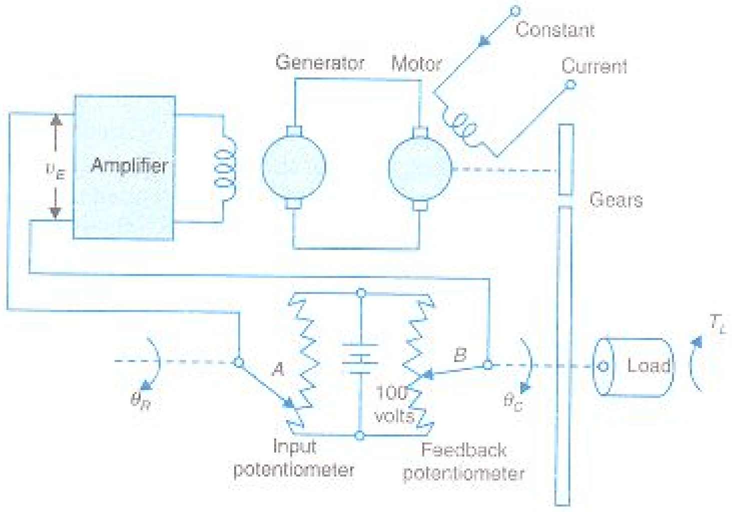


Fig 5.2. A position control system

By opening the feedback loop i.e., disconnecting the potentiometer B, it can easily be verified that any operator acting as part of feedback loop will find it very difficult to adjust θ_c to a desired value and to be able to maintain it. This further demonstrates the power of a negative feedback (hardware) loop.

The position control systems have innumerable applications, namely, machine tool position control, constant–tension control of sheet rolls in paper mills, control of sheet metal thickness in hot rolling mills, radar tracking systems, missile guidance systems, inertial guidance roll stabilization of sheets etc.

5.4 DC CLOSED-LOOP CONTROL SYSTEM

Figure below shows a typical dc (un-modulated) control system. The output signal θ_y represents the actual load position and the reference input θ_r represents the desired position of the load. A potentiometer error detector is used. The electrical error signal proportional to the difference in the positions of the actual and desired positions is amplified by the dc amplifier and this output drives the dc motor which in turn through the gear box decides the position of the load. The signal are all un-modulated(i.e. dc).

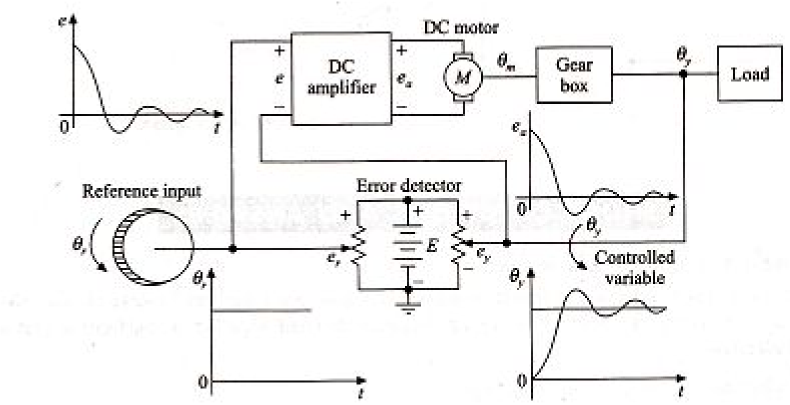
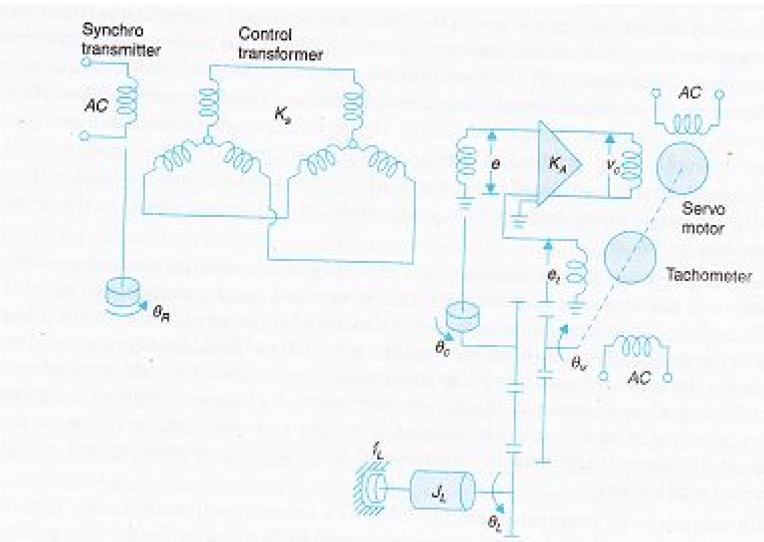
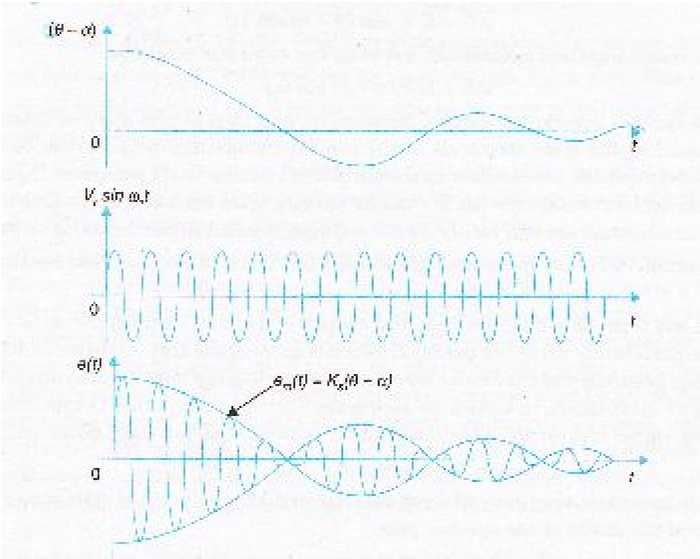


Fig.5.3, A DC close-loop control system

5.5 AC CLOSED-LOOP CONTROL SYSTEM

Figure below shows the schematic diagram of a typical ac control system. The output signal θ_y representing the actual load position is applied to the synchro control transformer. The reference input θ_r representing the desired output is applied to the synchro transmitter. The synchro pair acts as an error detector. The error signal is amplified by an ac amplifier and drives the ac servomotor which in turn positions the load through the gear box. The signals in this system are modulated (i.e. ac type).





[Fig.5.3, AC close-loop control system]

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Mathematical model of physical systems

CHAPTER-6

Introduction

A physical system is a collection of physical objects connected together to serve an objective. Examples of a physical system may be cited from laboratory, industrial plant or utility services an electronic amplifier composed of many components, the governing mechanism of a steam turbine or a communication satellite orbiting the earth are all examples of physical systems. A more general term *system* is used to describe a combination of components which may not all be physical, e.g., biological, economic, socio-economic or management systems.

No physical system can be represented in its full physical intricacies and therefore idealizing assumptions are always made for the purpose of analysis and synthesis of systems. An idealized physical model is called a *physical model.* a physical system can be modelled in a number of ways depending upon the specific problem to be dealt with and the desired accuracy. For example, an electronic amplifier may be modelled as an interconnection of linear lumped elements, or some of them may be pictured as non-linear elements in case the stress is on distortion analysis. A communication satellite may be modelled as a point, a rigid body or a flexible body depending upon the type of study to be carried out. As idealizing assumptions are gradually removed for obtaining more accurate model, a point of diminishing return is reached, i.e., the gain in accuracy of representation is not commensurate with the increased complexity of the computation required, in fact, beyond a certain point there may indeed be an undermined loss in accuracy of representation due to flow of errors in the complex computations.

Once a physical model of a physical system is obtained, the next step is to obtain a *mathematical model* which is the mathematical representation of the physical system through use of appropriate physical laws. Depending upon the choice of variables and the coordinate system, a given physical model may lead to different mathematical models. A network, for example, may be modelled as a set of nodal equations using Kirchhoff's current law or a set of mesh equations using Kirchhoff's voltage law. A control system may be modelled as a scalar differential equation describing the system or state variable vector-matrix differential equation. The particular mathematical model which gives a greater insight into the dynamic behaviour of physical system is selected.

When the mathematical model of a physical system is solved for various input conditions, the result represents the dynamic response of the system. The mathematical model of a system is *linear*, if it obeys the *principle of superposition* and *homogeneity*. This principle implies that if a system model has responses $y_1(t)$ and $y_2(t)$ to any two inputs $x_1(t)$ and $x_2(t)$ respectively, then the system response to the linear combination of these inputs

$\alpha_1 x_1(t) + \alpha_2 x_2(t)$ is given by the linear combination of the individual outputs i.e.,

$\alpha_1 y_1(t) + \alpha_2 y_2(t)$ Where α_1 and α_2 are constants.

Mathematical models of most physical systems are characterized by differential equations. A mathematical model is linear, if the differential equation describing it has coefficients, which are either functions only of the independent variables or are constants. If the coefficient of the differential equations is function of time (the independent variable) then the mathematical model is *linear time-varying*. On the other hand, if the coefficients of the describing differential equations are constants, the model is *linear time –invariant*.

The differential equation describing a linear time-invariant system can be reshaped into different forms for the convenience of analysis. For example, for transient response or frequency response analysis of single-input-single-output linear systems, the transfer function representation forms a useful model, On the other hand, when a system has multiple inputs and multiple outputs, the vector –matrix notation may be more convenient. The mathematical model of a system having been obtained, the available mathematical tools can then be utilized for analysis or synthesis of the system.

Powerful mathematical tools like the Fourier and Laplace Transforms are available for use in linear systems. Unfortunately no physical system in nature is perfectly linear. Therefore certain assumptions must always be made to get a linear model which is a compromise between the simplicity of the mathematical model and the accuracy of results obtained from it. However, it may not always be possible to obtain a valid linear model, for example, in the presence of a strong non-linearity or in presence of distributive effects which cannot be represented by lumped parameters.

A commonly adopted approach for handling a new problem is to first build a simplified model, linear as far as possible, by ignoring certain nonlinearities and other physical properties which may be present in the system and thereby get an approximate idea of the dynamic response; a more complete model is then built for more complete analysis.

DIFFERENTIAL EQUATIONS OF PHYSICAL SYSTEMS

This section presents the method of obtaining differential equation models of physical systems by utilizing the physical laws of the process. Depending upon the system well-known physical laws like Newton’s laws, Kirchhoff’s laws etc. will be used to build mathematical models.

We shall in first step build the physical model of the system as interconnection of idealized system elements and describe these in form of elemental laws. These idealized elements are sort of building blocks of the system. An ideal element results by making two basic assumptions.

1. Spatial distribution of the element is ignored and is regarded as a point phenomenon. Thus mass which has physical dimensions, is considered concentrated at a

point and temperature in a room which is distributed out into the whole room space is replaced by a representative temperature as if of a single point in the room.

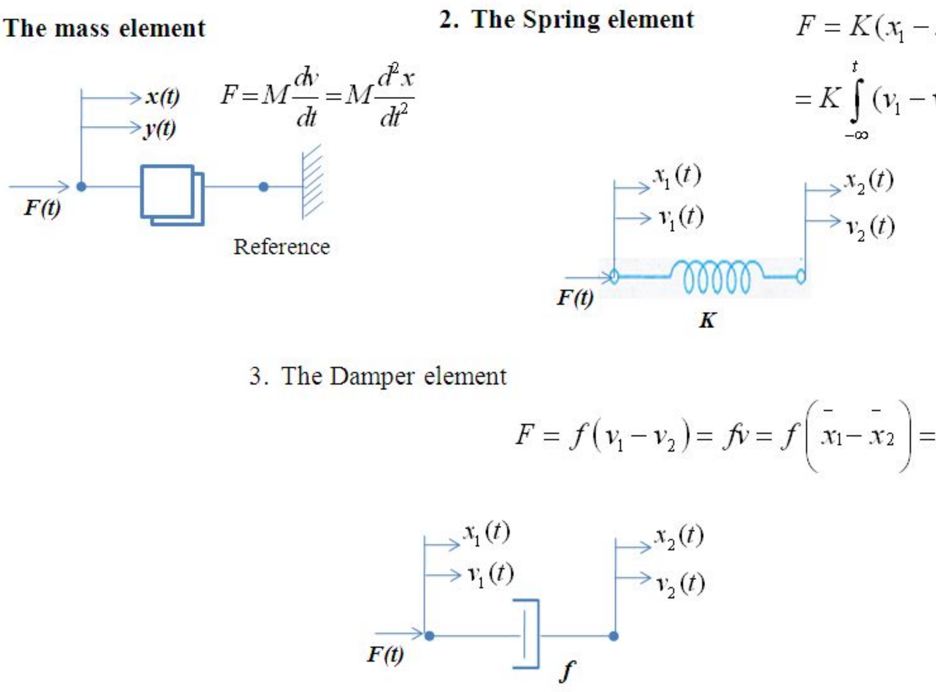
The process of ignoring the spatial dependence by choosing a representative value is called lumping and the corresponding modelling is known as lumped-parameter modelling as distinguished from the distributed parameter modelling which accounts for space distribution.

2. We shall assume that the variables associated with the elements lie in the range that the element can be described by simple inner law of (i) a constant of proportionality or (ii) a first order derivative or (iii) a first order integration.

The last two forms are in fact alternatives and can be inter-converted by a single differentiation or integration.

6.1 Mechanical systems

Mechanical systems and devices can be modelled by means of three ideal translator and three ideal rotary elements. Their diagrammatic representation and elemental relationships are given in Fig 6.1.In case of mass /inertia elements it may be noted that one terminal is always the inertial reference frame with respect to which the free terminal moves/rotates.



$$F = M\frac{dv}{dt} = M\frac{d^2x}{dt^2}$$

$$F = K(x_1 - x_2) = Kx = K\int_{-\infty}^{t}(v_1 - v_2)t = K\int_{-\infty}^{t} vdt$$

$$F = f(v_1 - v_2) = fv = f\left(\bar{x}_1 - \bar{x}_2\right) = f\bar{x}$$

$$x(m), v(m/\sec), M(kg), F(newton), K(newton/m), f(newton/m/\sec)$$

Fig 6.1, Ideal elements for mechanical translational systems

Translational systems

Let us consider a mechanical system shown in Fig6.2(a).It is simply a mass M attached to a spring(stiffness K) and a dashpot (viscous friction coefficient f) on which the force F acts. Displacement is positive in the direction shown. The zero position is taken to be at the point where the spring and mass are in static equilibrium.

Diagram

The schematic way of analyzing such a system is to draw a free-body diagram as shown in Fig.6.2(b)Then by applying Newton's law of motion to the free-body diagram ,the force equation can be written as

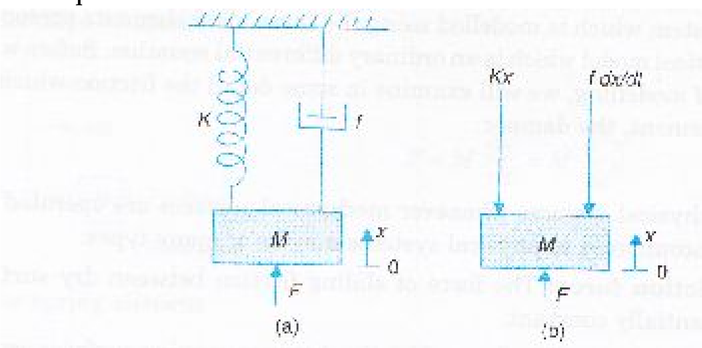


Fig.6.2

$$F - f\frac{dx}{dt} - Kx = = M\frac{d^2x}{dt^2}$$

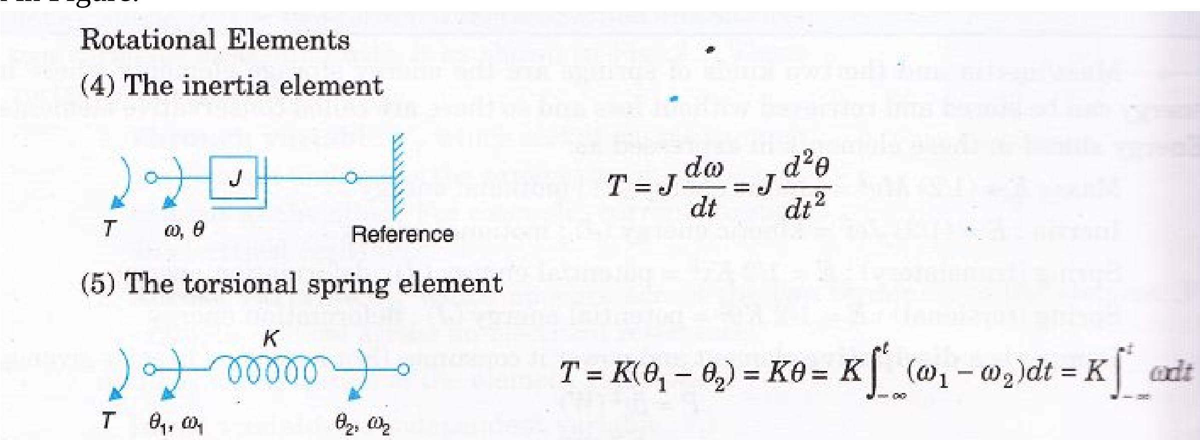
$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx \quad \text{...6.1}$$

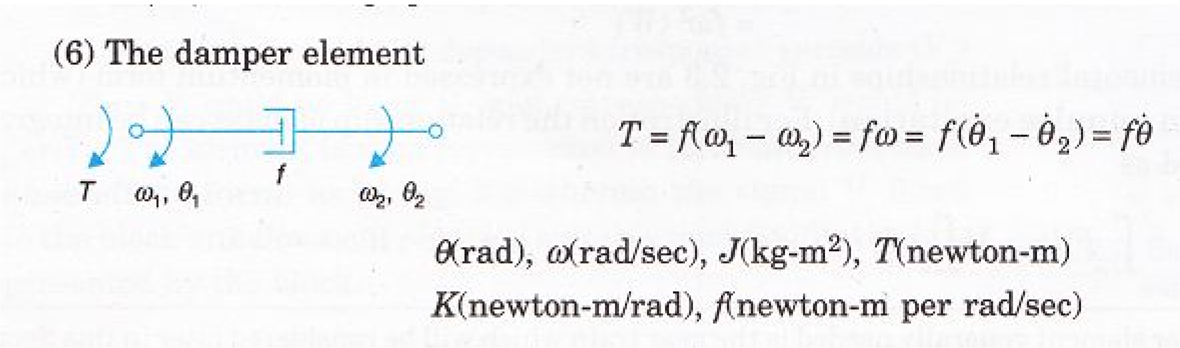
Equation (6.1) is a linear, constant coefficient differential equation of second-order. It may also be seen that there are two storage elements (mass M and spring K).

6.2 Rotational systems

Mechanical systems involving fixed-axis rotation occur in the study of machinery of many types and are very important. The modelling procedure is very close to that used in translation. In these systems, the variables of interest are the torque and angular velocity (or displacement).The three basic components for rotational systems are moment of inertia, torsion spring and viscous friction.

The three ideal rotational elements with their relevant properties and convention are shown in Figure.





[Fig 6.3, Ideal elements for mechanical rotational systems]

Let us consider now, the rotational mechanical system shown in Fig.6.2(a) which consists of a rotatable disc of moment of inertia J and shaft stiffness K. The disc rotates in a viscous medium with viscous friction coefficient f.

Let T be the applied torque which tends to rotate the disc. The free-body diagram is shown in Fig 6.2(b).

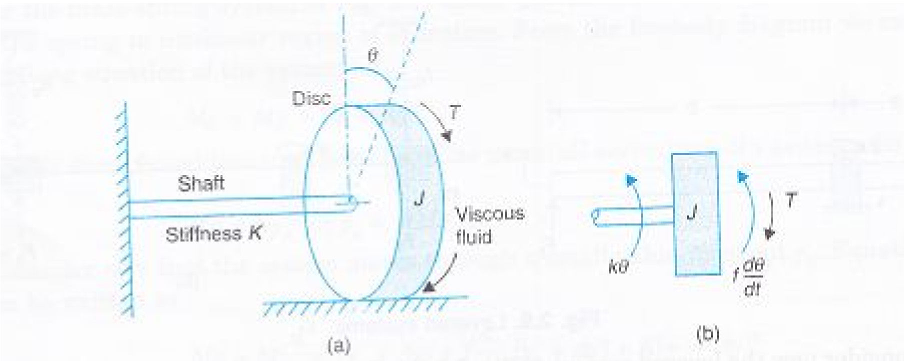


Fig 6.4.

The torque equation obtained from the free-body diagram is

$$T - f\frac{d\theta}{dt} - K\theta = J\frac{d^2\theta}{dt^2}$$

Or

$$T = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + K\theta \quad \text{...6.2}$$

6.3 Electrical systems

The resistor, inductor and capacitor are the three basic elements of electrical circuits. These circuits are analyzed by the application of Kirchhoff’s voltage and current laws.

Let us analyze the L-R-C series circuit shown in Fig.6.4 by using Kirchhoff’s voltage law. The governing equations of the system are

$$L\frac{di}{dt} + Ri + \frac{1}{c}\int_{\infty}^{t} i\,dt = e$$

$$\frac{1}{c}\int_{\infty}^{t} i\,dt = e_o \; ; \; e_o = e_c$$

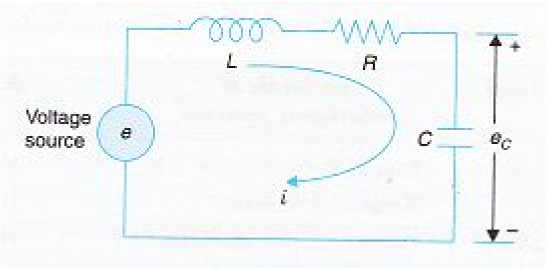


Fig . 6.5

Essential relationships are obvious from these equations. It is also to be noted that inductor and capacitor are the storage elements and resistor is the dissipative element. In terms of electric charge q = $\int i\,dt$, equation becomes

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{c}q = e \quad ....6.3$$

Similarly, using Kirchhoff's current law, we obtain the following equations for L-R-C parallel circuit fig.6.5.

$$C\frac{de}{dt} + \frac{1}{L}\int_{-\infty}^{t} edt + \frac{e}{R} = i \quad ....6.4$$

In terms of magnetic flux linkages $\emptyset = \int edt$, equation (2.25) may be written as $C\frac{d^2\Phi}{dt^2} + \frac{1}{R}\frac{d\Phi}{dt} + \frac{1}{L}\Phi = i$

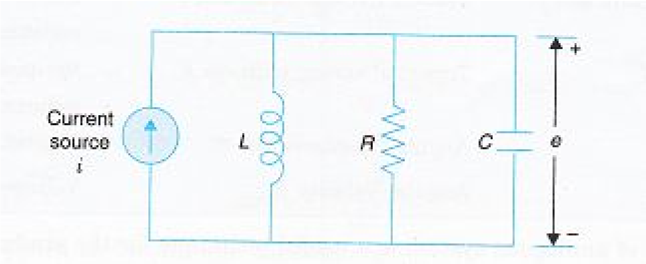


Fig.6.6

6.4 Analogous System

Comparing equation.(6.1) for the mechanical Translational system shown in Fig.6.2 or equation.(6.2) for the mechanical rotational system shown in Fig.6.4 and eqn.6.3 for the electrical system shown in Fig 6.5,it is seen that they are of identical form. Such systems whose differential equations are of identical form are identical forms are called analogous systems. The force F (torque T) and voltage e are the analogous variables here. This is called Force (Torque)-Voltage analogy.

Analogous Quantities in Force (Torque)-Voltage Analogy

Mechanical translational system	**Mechanical rotational system**	**Electrical system**
Force F	Torque T	voltage e
Mass M	Moment of inertia J	Inductance L
Viscous friction coefficient f	Viscous friction coefficient f	Resistance R
Spring stiffness K	Torsion spring stiffness K	Reciprocal of capacitance(1/c)
displacement x	Displacement θ	charge q

Velocity $\frac{dx}{dt}$	Angular Velocity $\frac{d\theta}{dt}$	current i

Similarly, equation 6.1 and 6.2 referred above and equation 6.4 for the electrical system shown in figure 6.6 are also identical. In this case force F [Torque T] and current i, are the analogous variables. This is called the Force (Torque)-current analogy. A list of analogous quantities in this analogy is given in Table .

Analogous Quantities in Force (Torque)-current Analogy:

Mechanical translational system	**Mechanical rotational system**	**Electrical system**
Force F	Torque T	current i
Mass M	Moment of inertia J	capacitance C
Viscous friction coefficient f	Viscous friction coefficient f	Reciprocal of Resistance (1/R)
Spring stiffness K	Torsion spring stiffness K	Reciprocal of Inductance (1/L)
Displacement x	Angular displacement θ	Magnetic flux linkage λ charge q
Velocity $\frac{dx}{dt}$	Angular Velocity $\frac{d\theta}{dt}$	voltage e

6.5 Transfer Functions

The transfer function of a linear time-invariant system is defined to be the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable under the assumption that all initial conditions are zero.

Consider the mass-spring-dashpot system shown in Fig 6.2, whose dynamics is described by the second order differential equation (6.1)

Taking the Laplace transform of each term of this equation (assuming zero initial conditions), we obtain

$$F(s) = Ms^2X(s) + f\,s\,X(s) + KX(s)$$

Then the transfer function is

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2+fs+K}$$

The highest power of the complex variable s in the denominator of the transfer function determines the *order of the system*. The mass-spring-dashpot system under consideration is thus a second-order system, a fact which is already recognised from its differential equation.

The transfer function of the L-R-C circuits shown in Fig.6.5 is similarly obtained by taking the Laplace transform of eqns.(6.3) & (6.4), with zero initial conditions. The resulting equations are

$$sLI(s) + RI(s) + \frac{1}{s}\frac{I(s)}{C} = E(s)$$

$$\frac{1}{s}\frac{I(s)}{C} = E_o(s)$$

If e is assumed to be the input variable and e_o the output variable, the transfer function of the system is

$$\frac{E_o(s)}{E(s)} = \frac{1}{LCs^2+RCs+1}$$

Above equations reveal that the transfer function is an expression in s-domain, relating the output and input of the linear time-invariant system in term of the system parameters and is independent of the input. It describes the input output behaviour of the system and does not give any information concerning the internal structure of the system. Thus, when the transfer function of a physical system is determined, the system can be represented by a block, which is a shorthand pictorial representation of the cause and effect relationship between input and output of the system. The signal flowing into the block(called input) flowing out of it (called output) after being processed by the transfer characterizing the block, see Fig 6.7(a).Functional operation of a system can be more readily visualized by examination of a block diagram rather than by the examination of the equations describing the physical system. Therefore, when working with a linear time-invariant system, we can think of a system or its sub-systems simply as interconnected blocks with each block described by a transfer function.

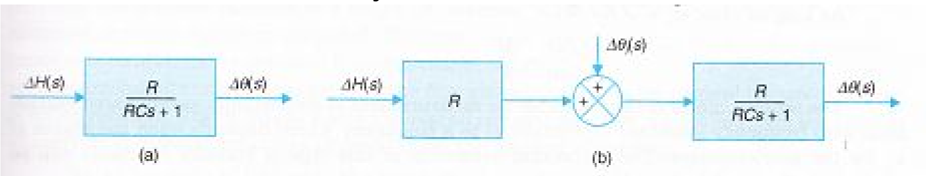


Fig.6.7

6.6 Transfer function of a single-input-single-output [SISO] system

The transfer function of a linear time in-variant system is defined as the Laplace transform of the impulse response ,with all the initial condition's set to zero. Let G(s) denote the transfer function of a single-input-single-output system, with input r(t) and output c(t) and impulse response g(t). Then the transfer function G(s) is defined as

$$G(s) = L[g(t)]$$

The transfer function G(s) is related to the Laplace transform of the input and the output through the following relation $G(s) = C(s)/R(s)$

With all initial condition set to zero, and C(s) and R(s) are the Laplace transforms of c(t) and r(t) respectively. That is the transfer function is defined as the ratio of Laplace transform of the output to the Laplace transform of the input with all initial conditions neglected.

Although the transfer function of a linear system is defined in terms of the impulse response, in practice, the input-output relation of linear time-invariant system with various data input is obtained described by a differential equation, so that it is more convenient to derive the transfer function directly from the differential equation. Let us consider that the input-output relation of a linear time-invariant system is described by the following nth order differential equation with constant real coefficients.

$$a_0 \frac{d^n c(t)}{dt^n} + a_1 \frac{d^{n-1} c(t)}{dt^{n-1}} + \dots\dots + a_{n-1} \frac{dc(t)}{dt} + a_n c(t) = b_0 \frac{d^m r(t)}{dt^m} + b_1 \frac{d^{m-1} r(t)}{dt^{m-1}} + \dots\dots + b_{m-1} \frac{dr(t)}{dt} + b_m r(t) \quad \dots 6.1a$$

Once the input r(t) for $t \geq t_0$ and the initial condition of c(t) and derivative of c(t) are specified at the initial time t= t_0 , the output response c(t0 for $t \geq t_0$ is determined by solving equation 6.1a. however, the solution of higher-order differential equation quite tedious . So the analysis and design of linear systems is done using transfer functions.

To obtain the transfer function of the linear system, simply take the Laplace transform on both sides of equation 6.1a and assume zero initial conditions.

The result is $(a_0 s^n + a_1 s^{n-1} + \dots\dots + a_{n-1} s + a_n)$ C(s) = $(b_0 s^m + b_1 s^{m-1} + \dots\dots + b_{m-1} s + b_m)$ R(s)

The transfer function between r(t) and c(t) is given by

$$G(s)= C(s)/R(s)= \frac{b_0 s^m + b_1 s^{m-1} + \dots\dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \dots\dots + a_{n-1} s + a_n}$$

6.7 CHARACTERISTICS EQUATION

The characteristics equation of a linear system is defined as the equation obtained by setting the denominator polynomial of the transfer function to zero. Thus, the characteristics equation of the system described above is:

$$a_0 s^n + a_1 s^{n-1} + \dots\dots + a_{n-1} s + a_n = 0$$

The above equation is called the characteristic equation because it characterizes the behaviour of the system.

The stability of linear single-input-single-output systems is governed completely by the roots of the characteristic equation.

6.8 Procedure for deriving transfer function

The following assumptions are made in deriving transfer functions of systems.

1. It is assumed that there is no loading, i.e. no power is drawn at the output of the system. If the system has more than one non-loading elements in cascade, then transfer function of each element can be determined independently. And the overall transfer function of the physical system is determined by multiplying the individual transfer functions. In case of systems consisting of elements which load each other, the overall transfer function should be derived by basic analysis without regard to individual transfer function.

2. The systems should be approximated by linear, lumped, constant parameter models by making suitable assumption.

To illustrate point 1 above, let us consider two identical RC circuits connected in cascade so that the output from the first circuit is fed as input to the second as shown in Figure of the electrical circuit.

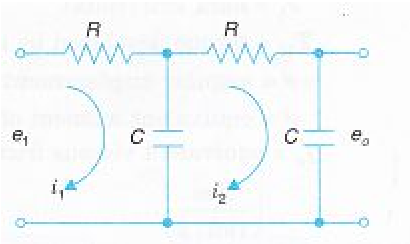


Fig.6.8

The describing equations for this system are as follows:

$\frac{1}{C}\int_{-\infty}^{t}(i_1 - i_2)\,dt + R\,i_1 = e_1$ 6.1a

$\frac{1}{C}\int_{-\infty}^{t}(i_2 - i_1)\,dt + R\,i_2 = -\frac{1}{C}\int i_2\,dt$ 6.1b

Taking the Laplace transform of equation 6.1a .and 6.1b , assuming zero initial conditions, we obtain

$\frac{1}{Cs}[I_1(s) - I_2(s)] + R\,I_1(s) = E_1(s)$

$\frac{1}{Cs}[I_2(s) - I_1(s)] + R\,I_2(s) = -\frac{1}{Cs}I_2(s) = -E_o(s)$ 6.2b

Reorganising equation (6.2b) , we get

$I_1(s)[R+\frac{1}{Cs}] = E_i(s) + \frac{1}{Cs}I_2(s)$

Or $I_1(s) = \frac{E_i(s) + \frac{1}{Cs}I_2(s)}{[R+\frac{1}{Cs}]} = \frac{Cs\,E_i(s) + I_2(s)}{1+RCs}$ 6.3a

Reorganising equation (6.3a) , we get

$I_2(s)[R+\frac{1}{Cs}] - \frac{1}{Cs}I_1(s) = -E_o(s)$

Substituting the values of I_1(s) and I_2(s) in the above equations , we get

Or $I_2(s)$)[R+$\frac{1}{Cs}$] - $\frac{Cs\,E_i(s) + I_2(s)}{Cs(1+RCs)}$ = $-E_o(s)$

Or

The transfer function is :

$\frac{E_o(s)}{E_i(s)} = \frac{1}{\tau^2 s^2 + 3\tau s + 1}$; where τ = RC.

****

Servomotors

CHAPTER-7

The control systems which are used to control the position or time derivatives of position, i.e. velocity and acceleration are called servo mechanisms. The motors which are used in automatic control systems are called servomotors. The servomotors are used to convert an electrical signal (control voltage) applied to them into an angular displacement of the shaft. Depending on the construction, they can operate either in a continuous duty or step duty.

A variety of servomotors are available for control system applications. The suitability of a motor for a particular application depends on the characteristics of the system, the purpose of the system and its operating conditions.

In general, a servomotor should have the following features:

1. Linear relationships between speed and electric control signal
2. Steady-state stability
3. Wide range of speed control
4. Linearity of mechanical characteristics throughout the entire speed range
5. Low mechanical and electrical inertia
6. Fast response

Servomotors are broadly classified as dc servomotors and ac servomotors depending on the power supply required to run the motor. Even though dc motors are costlier that ac motors, they have linear characteristics and so it is easier to control. They are generally used for large power applications such as in machine tool s and robotics.

The advantage of dc servomotors are as follows:

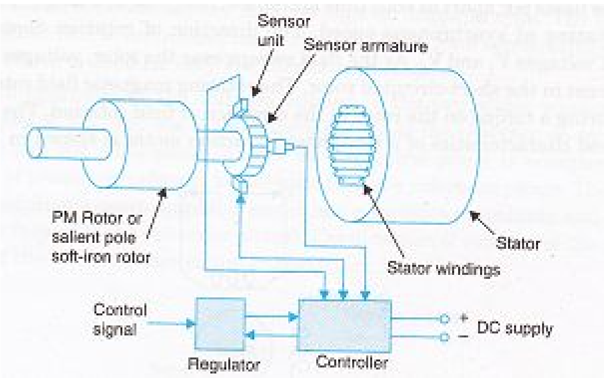
1. Higher output than for an ac motor of the same size.
2. Easy achievement of linear characteristics.
3. Easier speed control from zero speed to full speed in both the directions.
4. High torque to inertia ratio that gives them quick response to control signals.
5. The dc servomotors have light weight .low inertia and low inductance armature that can respond quickly to commands for a change in position or speed.
6. Low electrical and mechanical time constants.
7. The dc motor are capable of delivering over 3 times their rated torque for a short time compared to the 2 to 2.5 times the rated torque developed by the ac motors.

The advantage of ac servomotors are lower cost, higher efficiency and low maintenance since the brushes and commutators are not there. The disadvantages of ac servomotors are their characteristics are quite non linear and these motors are more difficult to control especially for positioning applications.

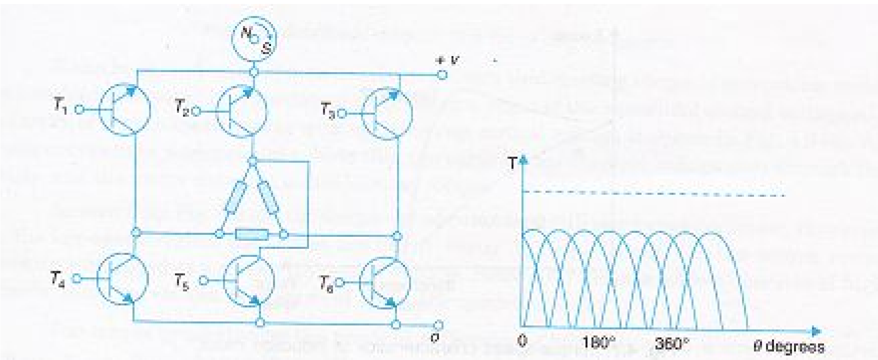
The ac motors are best suited for low power applications and computer equipments. The 3-phase induction motors with PWM power amplifier are currently gaining popularity in high power control applications.

7.1 DC SERVOMOTORS

The dc servomotors are broadly classified into (a) sliding contact motors with commutator and brushes and (b) brushless or contact less motors with SCR/Transistor commutator.



[Brushless dc servomotor]



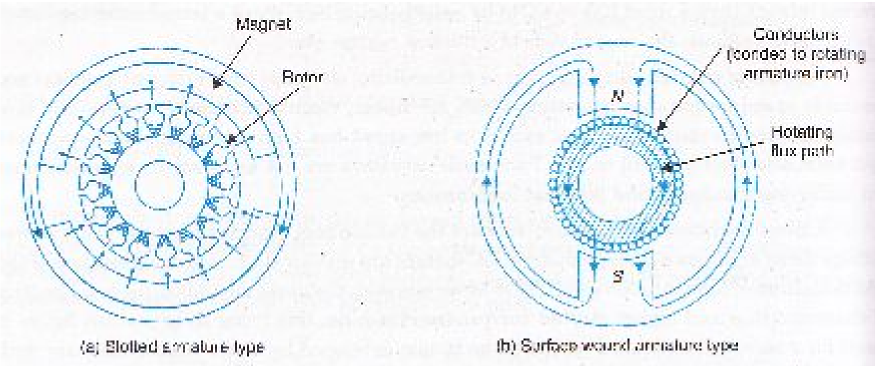
[3-phase,six-pulse brushless dc motor]

The sliding contact motors may be classified into (a) permanent magnet motors and (b) electro-magnetic field motors.

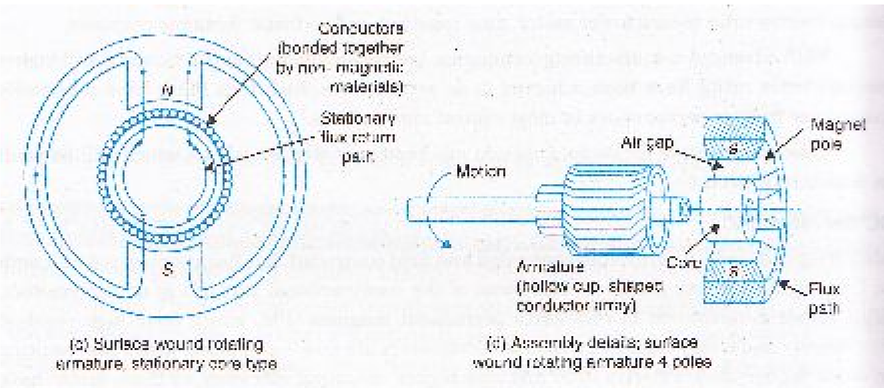
The permanent magnet motors mat be (a) cylindrical armature motors, or (b) disc armature motors, or (c) moving coil motors.

The electromagnetic field motors may be (a) armature-controlled motors, or (b) field-controlled motors, or (c) series motors, or split field motors.

PERMANENT MAGNET DC MOTORS



(a) Slotted armature type (b) Surface wound armature type



(c) Surface wound rotating armature, stationary core type (d) Assembly details; surface wound rotating armature 4 poles

In this type of motors, the armature is placed in rotor and the field winding is replaced by permanent magnet poles fixed to the stator to produce the required magnetic field. Permanent magnet motors are economical for power ratings up to a few kilowatts.

The following are some of the advantages of permanent magnet motors:

1. A simpler and more reliable motor because the field power supply is not required.
2. Higher operating efficiency as the motor has no field losses.
3. Field flux is less affected by temperature rise.
4. Higher torque/inertia ratio.
5. Speed is nearly directly proportional to the armature voltage at a given load torque.
6. A more linear torque/speed curve.
7. Higher output power at the same dimensions and temperature limitations.

The disadvantages of PM motors are: the magnets deteriorate with time and demagnetised with large transient currents. These drawbacks are eliminated by high grade materials such as ceramic magnets. But the cost of these materials is very high.

Electromagnetic field dc motors

Electromagnetic field motors are economical for higher power ratings generally above 1 KW. This type of servomotor is similar to a conventional dc motor construction with the following special features:

1. The number of slots and commutator segments is large to improve commutation.
2. Compoles and compensating windings are provided to eliminate sparking.
3. The diameter to length ratio is kept low to reduce inertia.
4. Over size shafts are employed to withstand the high torque stress.
5. Eddy currents are reduced by complete elimination of the magnetic circuit and by using low loss steel.

In this type of motor, the torque and speed may be controlled by varying the armature current and or field current. Generally one of these is varied to control the torque and the other is held constant. In armature-controlled mode of operation, the field current is held constant and the armature current is varied to control the torque. In the field control mode, the armature current is made constant and the field current is varied to control the torque.

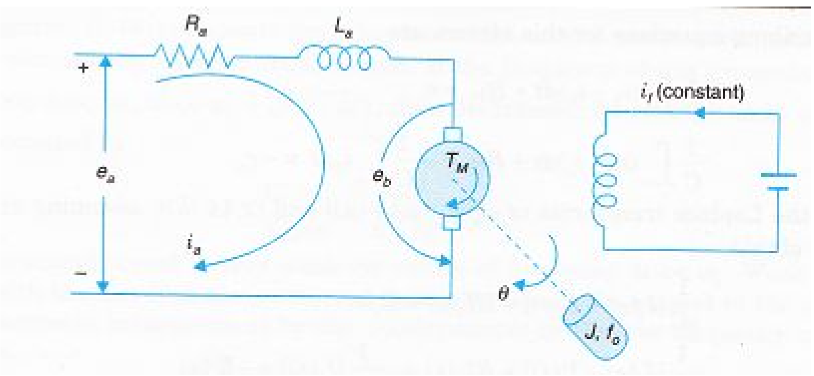
In servo applications, dc motors are required to produce rapid accelerations from standstill. Therefore, the physical requirements of such a motor are low inertia and high starting torque. Low inertia is attained with reduced armature diameter with a consequent increase in armature length such that the desired power output is achieved. Thus, except for minor differences in constructional features, a dc servomotor is essentially a ordinary dc motor.

Armature –controlled dc servomotor

An armature controlled dc servomotor is a dc shunt motor designed to satisfy the requirements of a servomotor. If the field current is constant, then speed is directly proportional to armature voltage, and torque is directly proportional to armature current. Hence the torque and speed can be controlled by armature voltage. Reversible operation is possible by reversing the armature voltage.

In small motors, the armature voltage is controlled by a variable resistance. But in large motors, in order to reduce power loss, armature voltage is controlled by Thyristors.

Figure below shows an armature-controlled dc motor.



In this system,
R_a = resistance of armature winding in ohms
L_a= inductance of armature winding in henrys
i_a= Armature current in amperes
i_f= Field current in amperes
E=applied armature voltage in volts
e_b= Back e.m.f in volts
T_M= torque developed by motor in N-m
θ = Angular displacement of motor shaft in radians
J= equivalent moment of inertia of motor and load referred to rotor shaft
f_o = equivalent viscous friction coefficient of motor and load referred to motor shaft.

In servo applications, the dc motors are generally used in linear range of the magnetisation curve. Therefore, the air –gap flux is proportional to the field current, i.e.

$\emptyset \propto i_f$
Or,
$\emptyset = K_f i_f$
Where K_f is a constant.

The torque T_M developed by the motor is proportional to the product of the air-gap flux $\emptyset$ and the armature currenti_a. i.e.

$$T_M \propto \Phi\, \mathbf{i}_a$$

$$T_M \propto K_f i_f\, \mathbf{i}_a$$

$$T_M = K_1 K_f i_f\, i_a$$

Where $K_1$is a constant.

In the armature –controlled dc motor, field current is kept constant, so the equation for T_M can be written as

$T_M = K_T i_a$

Where $K_1$is a constant known as motor torque constant.

The motor back emf is proportional to the speed i.e.

$e_b \propto \frac{d\theta}{dt}$ or $e_b = K_b \frac{d\theta}{dt}$ 7.1

Where K_b is a constant known as back emf constant .

The differential equation of the armature circuit is

$e = L_a \frac{di_a}{dt} + R_a i_a + e_b$ 7.2

The Torque equation is

$T_M = K_T i_a = J\frac{d^2\theta}{dt^2} + f_o \frac{d\theta}{dt}$ 7.3

Taking the Laplace transform of equation 7.1 .to 7.3,

$E_b(s) = K_b s\theta(s)$ 7.4

$E(s) = L_a s I_a(s) + R_a I_a(s) + E_b(s)$ 7.5

$i_a(s) = \frac{E(s)-E_b(s)}{(L_a s+R_a)}$

$Js^2\theta(s) + f_o s\,\theta(s) = T_M(s) = K_T I_a(s)$ 7.6

i.e. $\theta(s)(Js^2+f_o s) = K_T I_a(s)$

i.e. $s\theta(s)[Js+f_o] = K_T \frac{[E(s)-E_b(s)]}{(L_a s+R_a)} = K_T \frac{[E(s)-E_b(s)]}{(L_a s+R_a)}$

[Substituting the values of $I_a(s)$ and $E_b(s)$ from equations 7.4 and 7.5]

i.e. $(L_a s + R_a)s[Js+f_o]\,\theta(s) = K_T E(s) - K_b K_T.\ s\theta(s)$

$s[(L_a s + R_a)[Js+f_o] + K_b K_T]\,\theta(s) = K_T E(s)$

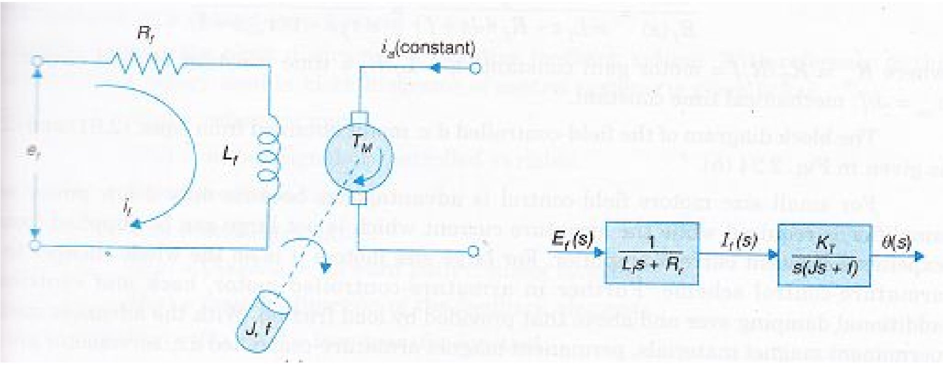
Therefore,the transfer function is

$$\frac{\theta(s)}{E(s)} = \frac{[E(s)-E_b(s)]}{s[(L_a s+R_a)[Js+f_o]+K_b K_T]}$$

Field–controlled dc servomotor :

A field controlled dc servomotor, is a dc shunt motor designed to satisfy the requirement of a servomotor. In this motor the armature is supplied with a constant current or voltage. When armature voltage is constant ,the torque is directly proportional to the field flux. Since the field current is proportional to the flux, the torque of the motor is controlled by controlling the field current. The response of a field controlled motor is however slowed by field inductance.

A field controlled dc motor is shown below.



In this system,

R_f = resistance of field winding in ohms

L_f= inductance of field winding in henrys

i_a= Armature current in amperes

i_f= Field current in amperes

e =applied field voltage in volts

e_b= Back e.m.f in volts

T_M= torque developed by motor in N-m

θ = Angular displacement of motor shaft in radians

J= equivalent moment of inertia of motor and load referred to rotor shaft

f_o = equivalent viscous friction coefficient of motor and load referred to motor shaft

In servo applications, the dc motors are generally used in linear range of the magnetisation curve. Therefore, the air –gap flux is proportional to the field current, i.e.

$\emptyset \propto i_f$

Or,

$\emptyset = K_f i_f$

Where K_f is a constant.

The torque T_M developed by the motor is proportional to the product of the air-gap flux $\emptyset$ and the armature current i_a. i.e.

$$T_M \propto \Phi\, i_a$$

$$T_M \propto K_f i_f\, i_a$$

$$T_M = K_T K_f i_f\, i_a = K_T' i_f$$

Where K_T' is a constant.

.

The differential equation of the armature circuit is

$e = L_f \frac{di_f}{dt} + R_f i_f$ 7.1a

The Torque equation is

$$T_M = K_T' i_f = J\frac{d^2\theta}{dt^2} + f_o \frac{d\theta}{dt}$$ 7.1b

Taking the Laplace transform of equation 7.1a .to 7.1b,

$E(s) = L_f s I_f(s) + R_f I_f(s)$

$i_f(s) = \frac{E(s)}{(L_f s + R_f)}$

$Js^2\theta(s) + f_o s\,\theta(s) = T_M(s) = K_T' i_f(s)$

i.e. $\theta(s)\,(Js^2 + f_o s) = T_M(s) = K_T' i_f(s) = \frac{K_T' E(s)}{(L_f s + R_f)}$

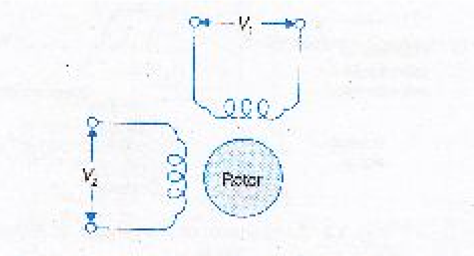
[Substituting the values of $I_f(s)$]

Therefore, the transfer function is

$$\frac{\theta(s)}{E(s)} = \frac{K_T'}{s(L_f s + R_f)(Js + f)}$$

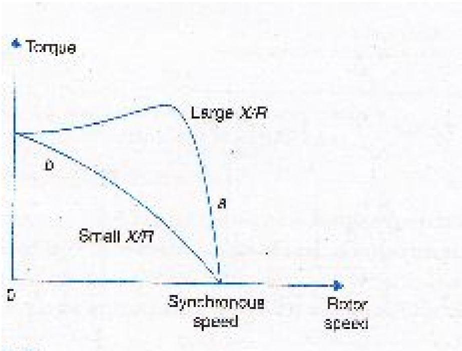
7.2 AC servomotors

An a.c. servomotor is basically a two-phase induction motor except for certain special design features. A two-phase induction motor consists of two stator windings oriented 90 degrees in Fig.7.1a shows the schematic diagram for balanced operation of the motor, i,e, voltages of equal r.m.s magnitude and 90 deg. Phase difference are applied to the two stator phases ,thus making their respective fields 90deg. Apart in both time and space, resulting in a magnetic field of constant magnitude rotating at synchronous speed. The direction of rotation depends upon phase relationship of voltages V1 & V2.As the field sweeps over the rotor, voltages are induced in it producing current in the short-circuited rotor. The rotating magnetic field interacts with these currents producing a torque on the rotor in the direction of field rotation. The general shape of the torque-speed characteristics of a two-phase induction motor is shown in Fig.7.1a



. Schematic diagram of a two-phase induction motor.

(Fig. 7.1a)



7. Torque-speed characteristics of induction motor.

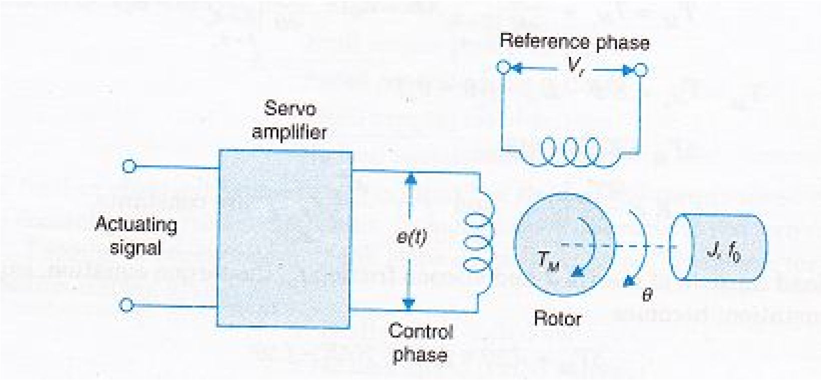
(Fig 7.1b)

It is seen from this figure that the shape of the characteristics depends upon the ratio of the rotor reactance X to the rotor resistance R. In normal induction motors, X/R ratio is generally kept high so as to obtain the maximum torque close to the operating region which is usually around 5% slip.

A two-phase servomotor differs in two ways from a normal induction motor.

1. The rotor of the servomotor is built with high resistance so that its X/R ratio is small and the torque-speed characteristics, as shown by the curve small b of Figure 7.1b is nearly linear in contrast to the highly non-linear characteristics when large X/R ration is used for servo applications, the because of the positive slope for part of the characteristics the system using such a motor becomes unstable.

The rotor construction is usually squirrel cage for drag-cup type .The diameter of the rotor is kept small in order to reduce inertia and thus to obtain good accelerating characteristics. Drag-cup construction is used for very low inertia applications.



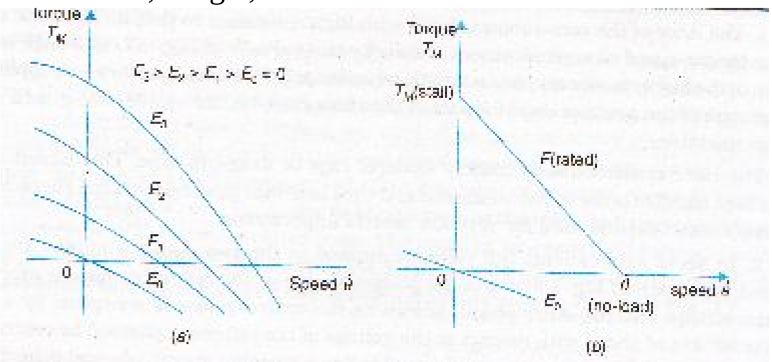
(Fig.7.1c)

2. In servo applications, the voltages applied to the two starter windings are seldom balanced. As shown in fig.7.1c, one of the phases known as the *reference phase* is excited by a constant voltage and the other phase, known as the *control phase* is energised by a voltage which is 90^0 out of phase with respect to the voltage of the reference phase. The control phase voltage is supplied from a servo amplifier and it has a variable magnitude and polarity ($\pm 90^0$phase angle w.r.t. reference phase). The direction of rotation of the motor reverses as the polarity of the control phase signal changes sign.

It can be proved using symmetrical components that starting torque of servo motor under unbalanced operation, is proportional to E, the r.m.s. value of the sinusoidal control voltage e(t).A family of torque –speed curves with variable r.m.s. control voltage is shown in Fig.7.2(a).All these curves have negative slope. Note that the curve for zero control voltage goes through the origin and the motor develops decelerating torque.

As seen from Fig.7.2 (a), the torque-speed curves are still somewhat non-linear. However, in the low speed region, the curves are nearly linear and equidistant, i.e., the torque varies linearly with speed as well as with control voltage. Since a servo motor seldom operates at high speeds, these curves can be linearized about the operating point.

The torque generated by the motor is a function of both the speed θ and r.m.s. control, voltage E,i.e.,$T_M = f(\theta, E)$. expanding this equation into Taylor's series about the normal operating point(T_{Mo} , E_o , θ_o) and droping of the terms of second –and higher order derivatives , we get,



(Fig.7.2)

$$T_M = T_{Mo} + \frac{\partial T_M}{\partial E}|E = E_o , \theta = \theta_o \ (E-E_o) + \frac{\partial T_M}{\partial \theta}|E = E_o , \theta = \theta_o \ (\theta - \theta_o) \text{ ....7.2a}$$

or

$$T_M - T_{Mo} = K(E-E_o) - f(\theta - \theta_o)$$

or

$$\Delta T_M = K\Delta E - f\ \Delta\theta \text{ ....7.2b}$$

where $K = \frac{\partial T_M}{\partial E}|E = E_o , \theta = \theta_o$

and

$f = - \frac{\partial T_M}{\partial \theta}|E = E_o , \theta = \theta_o (\theta - \theta_o)$

are constants.

If the load consists of inertia J and viscous friction f_o , the torque equation , expressed in incremental notation, becomes

ΔT_M =J $\Delta\theta$ + $f_o\Delta\theta$ = KΔE – f $\Delta\theta$

The above equation is valid even when ΔE varies with time so long as this variation is slow compared to $\sin\omega_c t$; in frequency domain ,it means that ω

(cut-off) $\ll$ ω_c where ω is the frequency of the signal ΔE , and ω_c is the carrier frequency. Control system using such devices is called *carrier control systems.*

7.3 Synchro Transmitter & Receiver

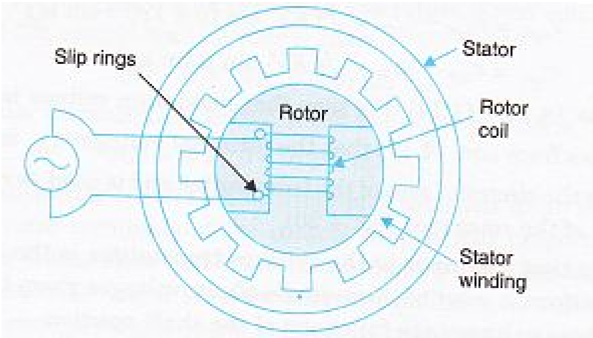
A synchro is an electro-magnetic transducer commonly used to convert angular position of a shaft into an electrical signal. It is commercially known as selsyn or autosyn.

The basic synchro unit is usually called a synchro transmitter. Its construction is similar to that of a three-phase alternator. The stator(stationary member) is of laminated silicon steel and is slotted to accommodate a balanced three phase winding which is usually of concentric coil type(three identical coils are placed in the stator with their axes 120 deg. Apart) and is Y-connected. The rotor is of dumbbell construction and is wound with a concentric coil. An ac voltage is applied to the rotor winding through slip rings. The constructional features and schematic diagram of a synchro transmitter are shown in Fig.7.3a and 7.3b respectively.

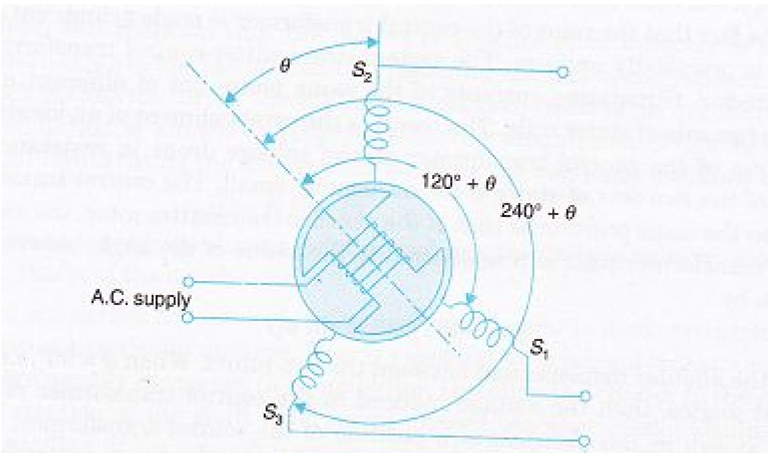
Let an ac voltage

$v_r(t) = V_r \sin\omega_c t$

Be applied to the rotor of the synchro transmitter as shown in Fig.4.18.This voltage causes a flow of magnetizing current in the rotor coil which produces a sinusoid ally time varying flux directed along its axis and distributed nearly sinusoid ally in the air gap along the stator periphery. Because of transformer action, voltages are induced in each of the stator coils. As the air gap flux is sinusoid ally distributed, the flux linking any stator coil is proportional to the cosine of the angle between the rotor and stator coil axes and so is the voltage induced in each stator coil. The stator coil voltages are of course in time phase with each other. Thus we see that the synchro transmitter acts like a single-phase transformer in which the rotor coil is the primary and the stator coils form the three secondary.



(a)



(b)

(Fig 7.3)

Let v_{s1n} , v_{s2n} and v_{s3n} respectively be the three voltages in the stator coils S_1 , S_2 and S_3 with respect to the neutral. Then, for the rotor position of the synchro transmitter shown in figure where the rotor axis makes an angle θ with the axis of the stator coil S_2 ,

$$v_{s1n} = K\, v_r \sin \omega_c t \cos(\theta + 120^0)$$

$$v_{s2n} = K\, v_r \sin \omega_c t \cos\theta$$

$$v_{s3n} = K\, v_r \sin \omega_c t \cos(\theta + 240^0)$$

The three terminal voltages of the stator are as follows:

$$v_{s1s2} = v_{s1n} - v_{s2n} = \sqrt{3}\, K\, v_r \sin(\theta + 240^0) \sin \omega_c t$$

$$v_{s2s3} = v_{s2n} - v_{s3n} = \sqrt{3}\, K\, v_r \sin(\theta + 120^0) \sin \omega_c t$$

$$v_{s3s1} = v_{s3n} - v_{s1n} = \sqrt{3}\, K\, v_r \sin\theta \sin \omega_c t$$

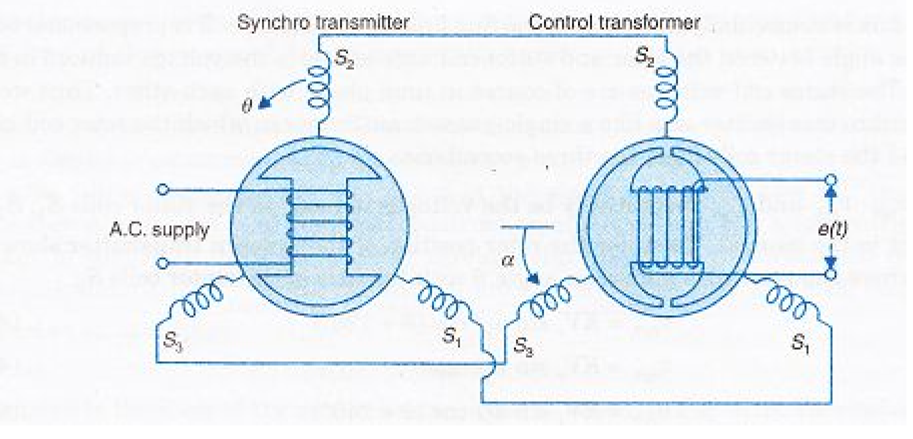
When $\theta = 0$

, from the above equations it is seen that maximum voltage is induced in the stator coil S_2 , while the terminal voltage v_{s3s1} =0.This position of the rotor is defined as the electrical zero of the transmitter and is used as reference for specifying the angular position of the rotor.

Thus it is seen that the input to the synchro transmitter is the angular position of its rotor shaft and the output is a set of three stator coil to coil single-phase .The magnitudes of these voltages are functions of shaft position.

7.4 Synchro error detector

The synchro error detector is formed by interconnection of the synchro transmitter and the synchro control transformer as shown in figure. The stator leads of the transmitter are directly connected to the stator leads of the control transformer. The angular position of the transmitter rotor is the reference input and the rotor is excited by a single phase ac supply. The control transformer rotor is connected to a servomotor and to the shaft of the load, whose position is the desired output. Initially, the transmitter and the control transformer are assumed to be in their electrical zero position. The angular separation of both rotor axes in this position is 90^0 .The electrical zero position of a control transformer in a servo system is that position of the rotor for which the output voltage on the rotor winding ism zero with the transmitter in its electrical zero position.



When the transmitter rotor is excited, the rotor flux is set up and emf's are impressed on the stator coils of the control transformer. The currents in the stator coils set up flux in the control transformer. Due to the similarity in the magnetic construction, the flux pattern produced in the two synchro's will be the same if all losses are neglected.

****

BLOCK DIAGRAM OF CONTROL SYSTEM

CHAPTE R- 8

INTRODUCTION

The input output behaviour of a linear system or element of a linear system is given by its transfer function G(s) = C(s)/R(s).

Where R(s) = Laplace transformation of the input variable, and C(s) = Laplace transform of the output variable.

A convenient graphical representation of this behaviour is the *block diagram* as shown in Fig.8.1 (a).Where in signal into the block represents the input R(s) and the signal out of the block represents the output C(s),while the block itself stands for the transfer function G(s).The flow of information(signal) is unidirectional from the input to the output with the output being equal to the input multiplied by the transfer function of the block.A complex system comprising of several non-loading elements is represented by the interconnection of the blocks for individual elements.The blocks are connected by lines with arrows indicating the unidirection flow of information from the output of one block to the input of the other. In addition to this, summing or differencing of signal is indicated by the symbols shown in fig.8.1(b), while the take-off point of a signal is represented by fig.8.1(c).

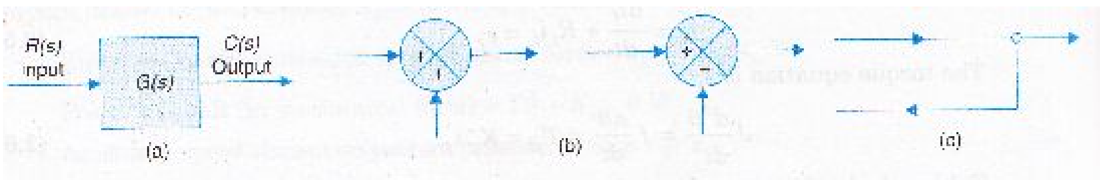


Fig.8.1

Block diagrams of some of the control systems turn out to be very complex such that the evaluation of their performance requires simplification (or reduction) of block diagrams which is carried out by block diagram rearrangements.

8.1 BLOCK DIAGRAM OF A CLOSED LOOP SYSTEM

The block diagram of a negative feedback system with reference to this Fig.8.1,the terminology used in block diagrams of a control system is given below.

R(s) = Reference input

C(s) = output signal or control variable

B(s) = Feedback signal

E(s) = Actuating signal

G(s) = C(s)/E(s) = Forward path transfer function

H(s) = Transfer function of the feedback elements

G(s) H(s) = B(s)/E(s) = Loop transfer function

T(s) = C(s)/R(s) = Closed-loop transfer function

From fig. 8.1 we have

C(s) = G(s)/E(s) 8.1

E(s) = R(s) – B(s) = R(s) – H(s) C(s) 8.2

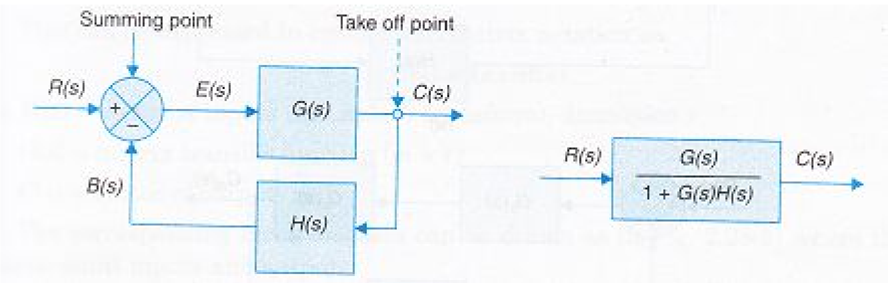


Fig.8.1

Eliminating E(s) from equation (8.1) and (8.2) we have

C(s) = G(s)R(s)-G(s)H(s)C(s)

or

C(s)/R(s) = T(s) = $\frac{G(s)}{1+G(s)H(s)}$ 8.3

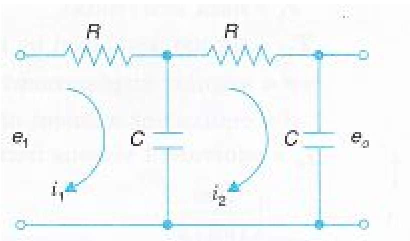
Therefore the system shown in Figure. 8.1 can be reduced to a single block shown in Fig.8.1.

8.2 Deriving transfer function

The following assumptions are made in deriving transfer functions of systems.

3. It is assumed that there is no loading, i.e. no power is drawn at the output of the system. If the system has more than one non-loading elements in cascade, then transfer function of each element can be determined independently. And the overall transfer function of the physical system is determined by multiplying the individual transfer functions. In case of systems consisting of elements which load each other, the overall transfer function should be derived by basic analysis without regard to individual transfer function.
4. The systems should be approximated by linear, lumped, constant parameter models by making suitable assumption.

To illustrate point 1 above, let us consider two identical RC circuits connected in cascade so that the output from the first circuit is fed as input to the second as shown in Figure of the electrical circuit.



The describing equations for this system are as follows:

$\frac{1}{c}\int_{-\infty}^{t}(i_1 - i_2)\,dt + R\,i_1 = e_1$ 8.3

$\frac{1}{c}\int_{-\infty}^{t}(i_2 - i_1)\,dt + R\,i_2 = -\frac{1}{c}\int i_2\,dt$ 8.4

Taking the Laplace transform of equation 8.3.and 8.4 , assuming zero initial conditions, we obtain

$\frac{1}{Cs}$ $[I_1(s) - I_2(s)] + R\, I_1(s) = E_1(s)$ 8.5

$\frac{1}{Cs}$ $[I_2(s) - I_1(s)] + R\, I_2(s) = -\frac{1}{Cs} I_2(s) = -E_o(s)$ 8.6

Reorganising equation (8.5) , we get

$I_1(s)[R+\frac{1}{Cs}] = E_i(s) + \frac{1}{Cs} I_2(s)$

Or $I_1(s) = \frac{E_i(s) + \frac{1}{Cs} I_2(s)}{[R+\frac{1}{Cs}]} = \frac{Cs\, E_i(s) + I_2(s)}{1+RCs}$

Reorganising equation (8.6) , we get

$I_2(s)[R+\frac{1}{Cs}] - \frac{1}{Cs} I_1(s) = -E_o(s)$

Substituting the values of $I_1(s)$ and $I_2(s)$ in the above equations , we get

Or $I_2(s))[R+\frac{1}{Cs}] - \frac{Cs\, E_i(s) + I_2(s)}{Cs(1+RCs)} = -E_o(s)$

Or

The transfer function is :

$\frac{E_o(s)}{E_i(s)} = \frac{1}{\tau^2 s^2 + 3\tau s + 1}$; where τ = RC.

8.3 Procedure for drawing Block diagram

A control system can be represented diagrammatically by a block diagram. To draw a block diagram for a system, first write the differential equation that describe the dynamic behaviour of each component. Then take the Laplace transforms of this differential equation assuming zero initial conditions to obtain the algebraic equations. These equations will have variables and constants. From the working knowledge of the system, the input and output variables are identified and the block diagram for each equation can be drawn. Each equation gives one section of block diagram; the output of one section will be the input for another section .The various sections are interconnected to obtain the overall block diagram of the system.

As an example, consider the RC circuit shown in Figure below. The equations for the circuit are as follows:

$$i = \frac{e_i - e_o}{R} \quad \text{.......................................8.1}$$

$$e_o = \frac{1}{C}\int i\,dt \quad \text{........................8.2}$$

The Laplace transform of equs. 8.1 and 8.2 with zero initial conditions yields ,

$$I(s) = \frac{E_i(s) - E_o(s)}{R} \quad \text{.............................8.3}$$

$$E_o(s) = \frac{1}{Cs} I(s) \quad \text{................8.4}$$

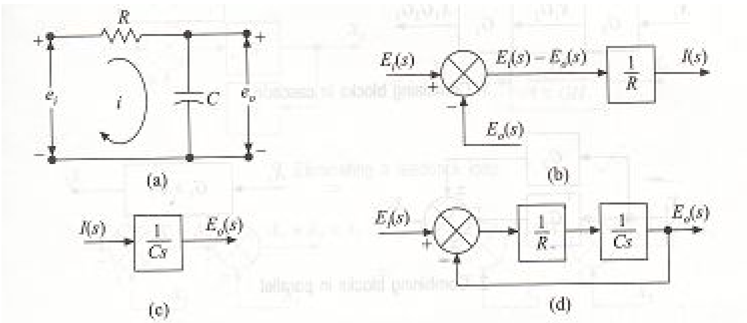


Fig 8.4

Equations 8.3 represent a summing operation and the corresponding diagram is shown in figure 8.4. Represent the block as shown in figure 8.4(c). Assembling these two elements, i.e. I(s) and $E_o(s)$ we obtained the overall block diagram of the system as shown in figure 8.4(d).

8.4 BLOCK DIAGRAM REDUCTION

A complex block diagram configuration can be simplified by certain rearrangements of block diagram using the rules of block diagram algebra. Some of the important rules are given in Table 8.5 . All these rules are derived by simple algebraic manipulation of the equations representing the blocks.

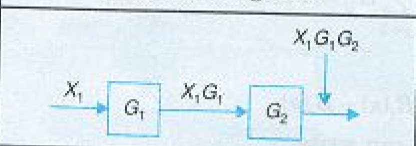
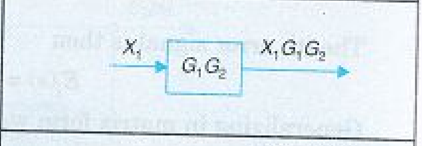
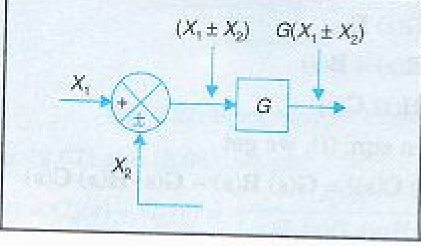
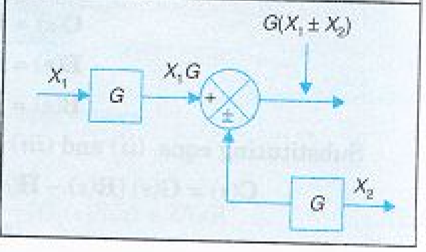
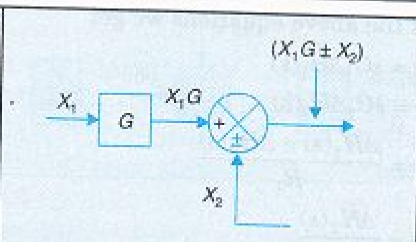
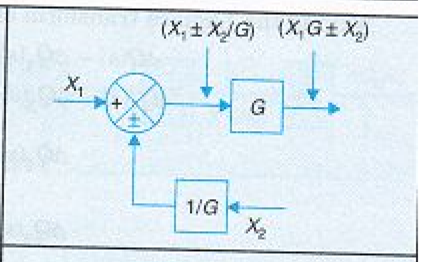
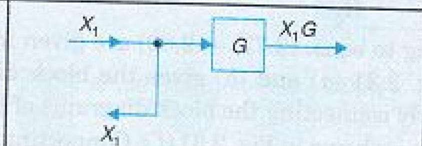
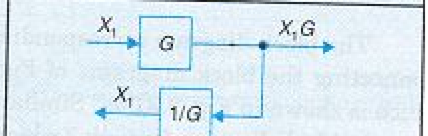
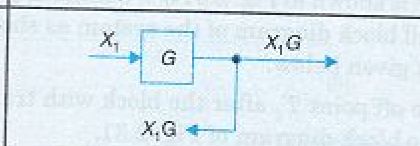
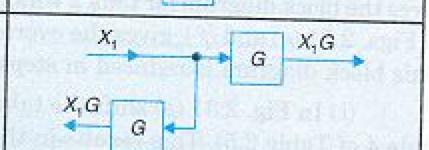
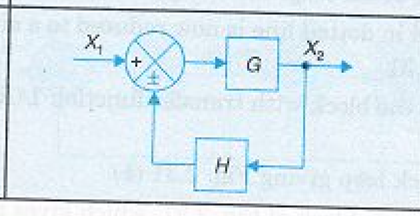
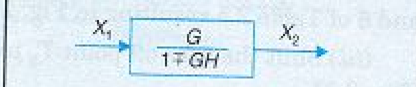
Rule	Original diagram	Equivalent diagram
1. Combining blocks in cascade		
2. Moving a summing point after a block		
3. Moving a summing point ahead of a block		
4. Moving a take off point after a block		
5. Moving a take off point ahead of a block		
6. Eliminating a feedback loop		

Table 8.5

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STABILITY OF CONTROL SYSTEM

CHAPTE R- 9

9.1 DEFINITION OF STABILITY OF CONTROL SYSTEM

Stability of a control system implies that small changes in the system input, in initial conditions or in system parameters, do not result in large changes in system output. Almost every working system is designed to be stable within permissible variations of the above changes.

A linear time-invariant system is stable if the following two notions of system stability are satisfied:

(i) When the system is excited by a bounded input, the output is bounded.
(ii) In the absence of the input, the output tends towards zero (the equilibrium state of the system) irrespective of initial conditions. **This concept of stability is known as asymptotic stability.**

The first notion concerns a system under the influence of an input. Clearly, if a system is subjected to an unbounded input and produces an unbounded response, nothing can be said about its stability. But if it is subjected to a bounded input and produces an unbounded response, it is by definition, unstable.Actually the output of an unstable system may increase to a certain extent and then the system may break down or become nonlinear after output exceeds a certain magnitude, so that the linear mathematical model no longer applies. The Second notion of stability generally concerns a free system relative to its transient behaviour.

Let us observe the physical implication of the two notions of stability defined earlier, by considering the single-input, single-output (SISO) system with transfer function

$$\frac{C(s)}{R(s)} = G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_n} \; ; \; m < n \quad \text{...9.1}$$

With initial conditions assumed zero, the output of the given system is given by

$$C(t) = \mathcal{L}^{-1}[G(s)R(s)]$$

Therefore

$$C(t) = \int_0^{\infty} g(\tau) r(t-\tau)\, d\tau$$

Where $g(t) = \mathcal{L}^{-1}[G(s)]$ is the impulse response of the system

Taking the absolute value of both sides we get

$$|\, C(t)\, | = |\, \int_0^{\infty} g(\tau) r(t-\tau)\, d\tau \,|$$

Since the absolute value of integral is not greater than the integral of the absolute value of the integrand,

$$|\, C(t)\, | \leq \int_0^{\infty} |\, g(\tau) r(t-\tau)\, |\, d\tau \,| \leq \int_0^{\infty} |\, g(\tau)\, ||\, r(t-\tau)\, |\, d\tau \,| \quad \textbf{..................9.2}$$

The first notion of stability is satisfied if for every bounded input (| **r(t)** | $\leq M_1 < \infty$ **),** the output is bounded (| C(t) | $\leq M_2 < \infty$ **)**. From equ.9.2 we have for bounded input, the bounded output condition as

$$|\, C(t)\, | \leq M_1 \int_0^{\infty} |\, g(\tau)\, |\, d\tau \,| < M_2$$

Thus the first notion of stability is satisfied if the impulse response g(t) is absolutely integrable, i.e. $\int_0^{\infty} |\, g(\tau)\, |\, d\tau$ is finite area under the absolute-value curve of the impulse response g(t) evaluated from t=0 to t= ∞ must be finite).

The nature of g(t) is dependent on the poles of the transfer function G(s) which are the roots of the characteristics equation. These roots may be both real and complex conjugate and may have multiplicity of various orders.

9.2 NECESSARY CONDITIONS FOR STABILITY

Certain conclusions regarding the stability of a system can be drawn by merely inspecting the coefficients of its characteristics equation in polynomial form. In the following sections, we shall show that necessary (but not sufficient) condition for stability of a linear system is that all the coefficients of its characteristics equation q(s) = 0, be real and have the same sign. Furthermore, none of the coefficients should be zero.

Consider the characteristics equation

$$q(s) = a_0 s^n + a_1 s^{\square-1} + a_2 s^{n-2} + \dots\dots\dots\dots + a_{n-1}\square + a_n = 0 \; ; \; a_0 > 0 \quad \text{....9.1a}$$

It is to be noted that there is no loss of generality in assuming $a_0 > 0$ .In case $a_0 < 0$; it can be made positive by multiplying the characteristics equation by -1 throughout.

Equation (9.1a) may be written in the factored form as

$$q(s) = a_0 \prod (s + s_k) \; \prod[(s + \sigma_l)^2 + \omega_l^2] \quad \text{...............................9.1b}$$

For the system to be stable, the roots should have negative real parts, which is satisfied if in equation (9.1b) all s_k and σ_l are positive real. It means that all the factors of equation (9.1b) have positive terms only. As these factors are multiplied together to get the characteristics equation in polynomial form, all the coefficients of the resulting polynomial must work out to positive. However, if one or more roots have positive real parts, the coefficients of the characteristics equation may or may not be all positive and hence the above conclusion.

Furthermore, it is to be noted that none of the coefficients can be zero or negative unless one (or more that one) of the following occurs:

(1) One or more roots have positive real parts

(2) A root (or roots) at origin, i.e.,s_k =0 and hence , $\square_n$ =0 ;

(3) σ_l =0 for some l, which implies the presence of roots on the j□-axis.

We therefore conclude that the absence or negative-ness of any of the coefficients of the characteristics equation (with $a_0 > 0$) indicates that the system is either unstable or at most limitedly stable.

From the foregoing discussions, let us prove the following propositions:

1. The positiveness of the coefficients of characteristics equation is necessary as well as sufficient condition for stability of system of first and second-order.

The characteristics equation of first order system is

$$a_0 s + a_1 = 0 \quad \text{...9.1c}$$

Which has a single root $s_1 = - a_1 / a_0$

It is obvious from eqn.(9.1c) that the positiveness of a_0 and a_1 ensures a negative root, i.e., stability.

The characteristics equation of a second-order system is

$a_0 s^2 + a_1 s + a_2 = 0$ which has the roots 9.1d

$$s_1, s_2 = [-a_1 \pm \sqrt{(a_1^2 - 4a_0 a_2)}]/2a_0$$

From equation (9.1d) it is seen that the positiveness of a_0, a_1 and a_2 ensures that the roots lie in left half of the s-plane(either both the roots are negative or they have negative real parts) which implies the stability of the system.

Therefore the first step in analyzing the stability of the system is to examine its characteristics equation and If some of the coefficients are zero or negative, it can be concluded that the system is not stable. On the other hand, if all the coefficients of the characteristics equation are positive (or negative), the possibility of stability of system exists and one should proceed further to examine the sufficient condition of stability.

9.3 ROUTH STABILITY CRITERION

This criterion is based on ordering the coefficients of the characteristics equation into an array, called the Routh array as given below:

$q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots\dots\dots + a_{n-1} s + a_n = 0$ 9.2

Routh array

s^n	a_0	a_2	a_4	a_6	.	.	
s^{n-1}	a_1	a_3	a_5	a_7	.	.	
s^{n-2}	b_1	b_2	b_3		.	.	.
s^{n-3}	c_1	c_2	.	.			
s^{n-4}	d_1	d_2	.	.			
	.	.					
	.	.					
s^2	e_1	a_n		.	.		
s^1	f_1	.	.				
s^0	a_n						

The coefficients b_1, b_2, ..., are evaluated as follows :

$b_1 = (a_1 a_2 - a_0 a_3)/ a_1$;

$b_2 = (a_1 a_4 - a_0 a_5)/ a_1$;

This process is continued till we get a zero as the last coefficient in the third row. In a similar way, the coefficients of 4^{th}, 5th,...nth and (n+a)th rows are evaluated, e.g.,

$c_1 = (b_1 a_3 - a_1 b_2)/ b_1$;

$$c_2 = (b_1 a_5 - a_1 b_3)/ b_1 \; ;$$

$$d_1 = (c_1 b_2 - b_1 c_2)/ c_1 \; ;$$

$$d_2 = (c_1 b_3 - b_1 c_3)/ c_1 \; ;$$

It is to be noted here that in the process of generating the Routh array, the missing terms are regarded as zero. Also all the elements of any row can be divided by a positive constant during the process to simplify the computational work.

The Routh stability criterion is stated as below:

For a system to be stable,it is necessary and sufficient that each term of first column of Routh array[as given in equn.(9.2)] of its characteristics equation be positive if $a_0 > 0$.If this condition is not met, the system is unstable and number of sign changes of the terms of the first column o9f the Routh array corresponds to the number of roots of the characteristics equation in the right half of the s-plane.

9.4 APPLICATION OF ROUTH STABILITY CRITERION TO LINEAR FEEDBACK SYSTEM

Example : Discuss the stability of the system with characteristics equation

$$s^4 + 8 s^3 + 18 s^2 + 16s + 5 = 0$$

Solution: The Routh array of this system is given below:

s^4	1	18	5
s^3	8	16	0(for the missing term)
s^2	$\frac{8\times18-1\times16}{8}$ =16	$\frac{8\times5-1\times0}{8}$ =5	
s^1	$\frac{16\times16-8\times5}{16}$ =135	0	
s^0	5		

The elements of the first column are all positive and hence the system is stable.

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